

Part 0: A gentle introduction to nonlinear optimization

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minimize $f(x)$ subject to $c_E(x) = 0$ and $c_I(x) \geq 0$
 $x \in \mathbb{R}^n$

MSc course on nonlinear optimization

WHAT IS NONLINEAR PROGRAMMING?

Nonlinear optimization \equiv **nonlinear programming**

minimize $f(x)$ subject to $c_{\mathcal{E}}(x) = 0$ and $c_{\mathcal{I}}(x) \geq 0$
 x

where

objective function $f : \mathbb{R}^n \longrightarrow \mathbb{R}$

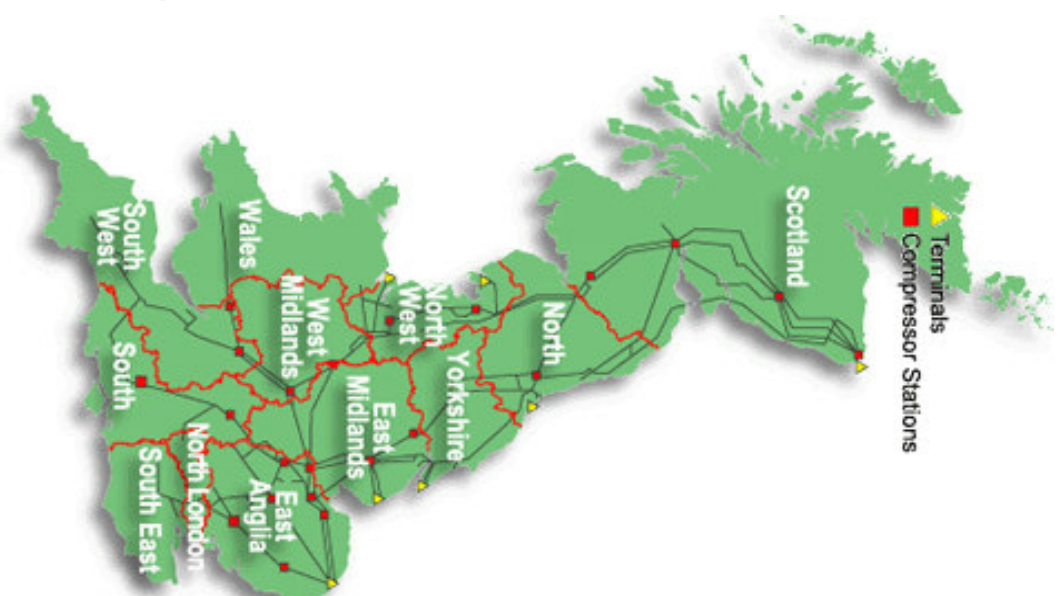
constraints $c_{\mathcal{E}} : \mathbb{R}^n \longrightarrow \mathbb{R}^{m_e}$ ($m_e \leq n$) and

$$c_{\mathcal{I}} : \mathbb{R}^n \longrightarrow \mathbb{R}^{m_i}$$

- there may also be integrality restrictions

AN EXAMPLE

Optimization of
a high-pressure
gas network



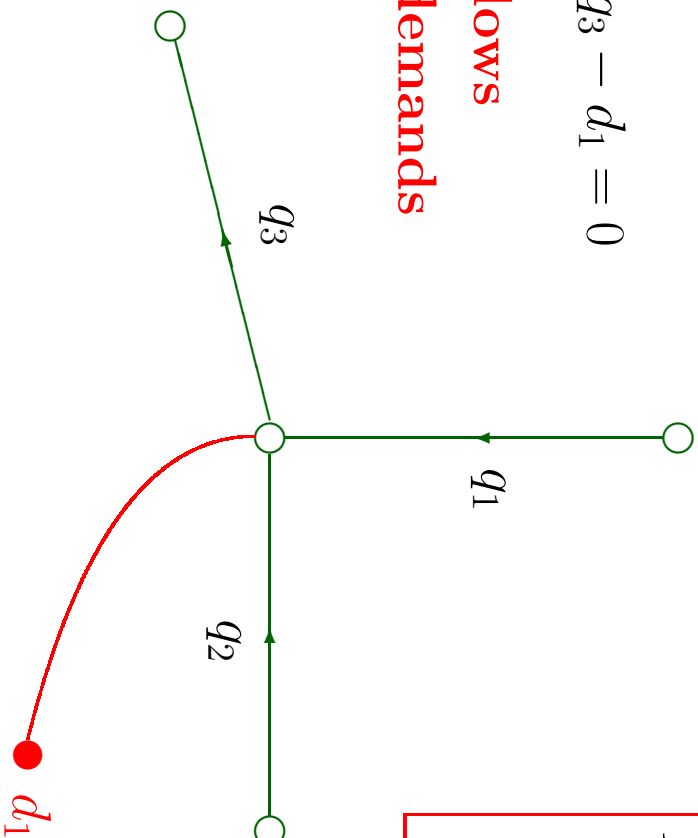
Transco
National
Transmission
System

British Gas (Transco)
Oxford University
RAL

NODE EQUATIONS

$$q_1 + q_2 - q_3 - d_1 = 0$$

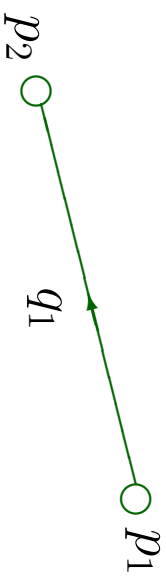
where q_i **Flows**
 d_i **demands**



In general: $Aq - d = 0$

- linear
- sparse
- structured

PIPE EQUATIONS



$$p_2^2 - p_1^2 + k_1 q_1^{2.8359} = 0$$

where p_i **pressures**

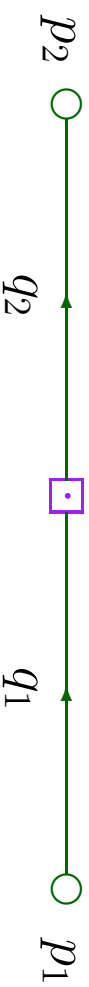
q_i **flows**

k_i **constants**

In general: $ATp^2 + Kq^{2.8359} = 0$

- non-linear
- sparse
- structured

COMPRESSOR CONSTRAINTS



$$q_1 - q_2 + z_1 \cdot c_1(p_1, q_1, p_2, q_2) \geq 0$$

where p_i **pressures**

q_i **flows**

z_i **0–1 variables**

= 1 if machine is on

c_i **nonlinear functions**

In general: $A_2^T q + z \cdot c(p, q) \geq 0$

- non-linear
- sparse
- structured
- 0–1 variables

OTHER CONSTRAINTS

Bounds on pressures and flows

$$p_{\min} \leq p \leq p_{\max}$$
$$q_{\min} \leq q \leq q_{\max}$$

- simple bounds on variables

OBJECTIVES

Many possible objectives

- maximize / minimize sum of pressures
 - minimize compressor fuel costs
 - minimize supply
- + combinations of these

STATISTICS

British Gas National Transmission System

- 199 nodes
- 196 pipes
- 21 machines

Steady state problem

~400 variables

24-hour variable demand problem with 10 minute discretization

~58,000 variables

Challenge: Solve this in real time

TYPICAL PROBLEM

This problem is typical of real-world, large-scale applications

- ◉ simple bounds
- ◉ linear constraints
- ◉ nonlinear constraints
- ◉ structure
- ◉ global solution “required”
- ◉ integer variables
- ◉ discretization

(SOME) OTHER APPLICATION AREAS

- minimum energy problems
- structural design problems
- traffic equilibrium models
- production scheduling problems
- portfolio selection
- parameter determination in financial markets
- hydro-electric power scheduling
- gas production models
- computer tomography (image reconstruction)
- efficient models of alternative energy sources

CLASSIFICATION OF OPTIMIZATION PROBLEMS

