CNAc: Continuous Optimization
Problem set 3 — trust-region methods

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Instructions: Asterisked problems are intended as a homework assignment., Please put your solutions in Denis Zuev's pigeon hole at the Maths Institute by 9AM on Monday of 6th week.

***Problem 1.**

Describe how you would find the Cauchy point for a second-order model

$$m_k(s) = f_k + s^T g_k + \tfrac{1}{2} s^T B_k s$$

of $f(x_k + s)$ within the trust-region $\|s\| \leq \Delta_k$.

***Problem 2.**

Solve the "trust-region" sub-problem

$$\underset{s \in \mathbb{R}^n}{\text{minimize}} \;\; s^T g + \tfrac{1}{2} s^T B s \;\text{ subject to }\; \|s\|_2 \leq \Delta \tag{1}$$

in the following cases:

(a)

$$B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}, \quad g = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \text{ and } \Delta = 2,$$

(b)

$$B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}, \quad g = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \text{ and } \Delta = 5/12$$

[Hint: a root of the nonlinear equation

$$\frac{1}{(1+\lambda)^2} + \frac{1}{(2+\lambda)^2} = \frac{25}{144}$$

is $\lambda = 2$.],

(c)

$$B = \begin{pmatrix} -2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad g = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \text{ and } \Delta = 5/12,$$

(d)

$$B = \begin{pmatrix} -2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad g = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \text{ and } \Delta = 1/2, \text{ and}$$

(e)

$$B = \begin{pmatrix} -2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad g = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \text{ and } \Delta = \sqrt{2}.$$

***Problem 3.**

Sketch the solution of problem (1) with data

$$B = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix} \text{ and } g = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

as a function of the trust-region radius Δ. In which direction does the solution point as Δ shrinks to zero? How does the Lagrange multiplier for the trust-region constraint depend on the trust-region radius? For what value of the radius does the solution become unconstrained?

***Problem 4.**

Suppose M is a symmetric, positive-definite matrix and we define the M-norm of a vector s so that $\|s\|_M^2 = s^T M s$. Find necessary and sufficient conditions for s_* to

$$\underset{s \in \mathbb{R}^n}{\text{minimize}} \;\; q(s) \equiv s^T g + \tfrac{1}{2} s^T B s \;\text{ subject to }\; \|s\|_M \leq \Delta.$$

[Hint: recall that any symmetric positive definite matrix M may be written as $M = R^T R$ for some non-singular R.]