Introduction to Continuous Optimisation
Dr R. Hauser – 16 HT

Aims: Optimisation deals with the problem of minimising or maximising a mathematical model of an objective function such as cost, fuel consumption etc. under a set of side constraints on the domain of definition of this function. Optimisation theory is the study of the mathematical properties of optimisation problems and the analysis of algorithms for their solution. The aim of this course is to provide an introduction to nonlinear continuous optimisation specifically tailored to the background of mathematics students.

Synopsis:

1 Preliminaries: convex sets and functions, Cholesky and QR factorisations, Sherman–Morrison–Woodbury formula, implicit function theorem, global versus local optimisation, convergence rates, optimality conditions for unconstrained optimisation.

2–4 Line-search methods for unconstrained optimisation: steepest descent, congujate gradients, Fletcher–Reeves method, Newton–Raphson method, symmetric rank one method, Broyden–Fletcher–Goldfarb–Shanno method, practical line searches.

5–7 Trust region methods for unconstrained optimisation: Cauchy point, dog-leg method, two dimensional subspace minimisation, Steihaug's method, characterisation of exact solutions.

8–11 Optimality conditions for constrained optimisation: convex separation, Farkas' lemma, linear programming duality, constraint qualification, Lagrangian function, Karush–Kuhn–Tucker conditions, second order optimality conditions, Lagrangian duality.

12–16 Nonlinearly constrained optimisation: merit functions and homotopy idea, penalty function method, augmented Lagrangian method, barrier method, sequential quadratic programming.

Text: Lecture notes will be made available for downloading from the course webpage. To complement the notes, reading assignments will be given from the book of J. Nocedal and S. J. Wright, *Numerical Optimisation*, Springer 1999.