Part 0: A gentle introduction to nonlinear optimization

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$$\underset{x \in \mathbb{R}^n}{\text{minimize}} \;\; f(x) \;\; \text{subject to} \;\; c_{\mathcal{E}}(x) = 0 \;\; \text{and} \;\; c_{\mathcal{I}}(x) \geq 0$$

MSc course on nonlinear optimization

WHAT IS NONLINEAR PROGRAMMING?

Nonlinear optimization $\equiv$ **nonlinear programming**

$$\underset{x}{\text{minimize}} \;\; f(x) \;\; \text{subject to} \;\; c_{\mathcal{E}}(x) = 0 \;\; \text{and} \;\; c_{\mathcal{I}}(x) \geq 0$$

where

objective function $f : \mathrm{IR}^n \longrightarrow \mathrm{IR}$

constraints $c_{\mathcal{E}} : \mathrm{IR}^n \longrightarrow \mathrm{IR}^{m_e}$ $(m_e \leq n)$ and
$c_{\mathcal{I}} : \mathrm{IR}^n \longrightarrow \mathrm{IR}^{m_i}$

⊙ there may also be integrality restrictions

AN EXAMPLE

Optimization of
a high-pressure
gas network

Transco
National
Transmission
System

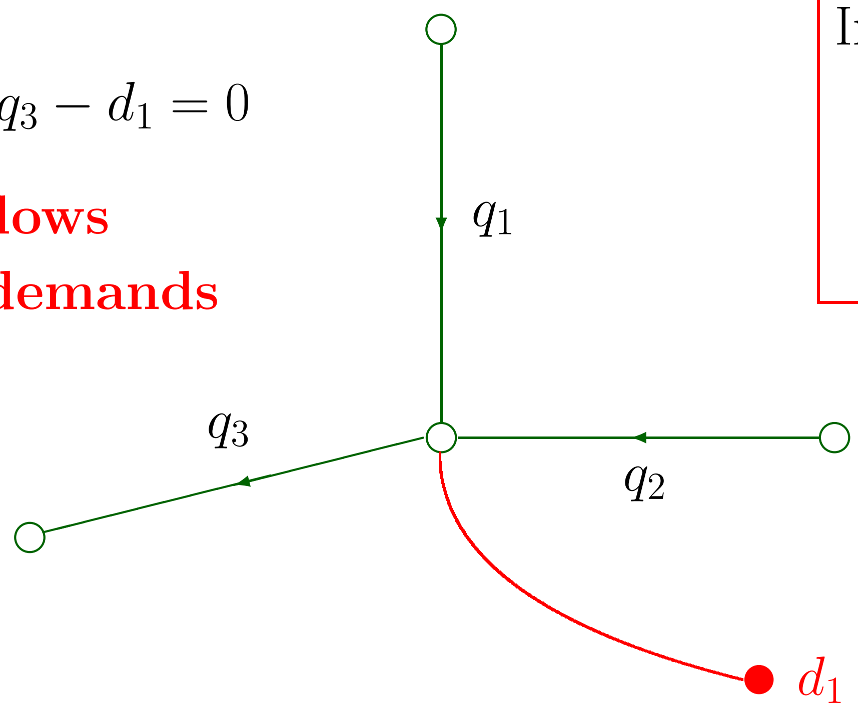
British Gas (Transco)
Oxford University
RAL

NODE EQUATIONS

$$q_1 + q_2 - q_3 - d_1 = 0$$

where q_i **flows**

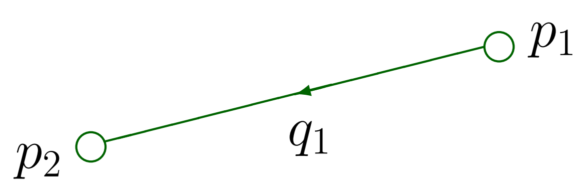
d_i **demands**



In general: $Aq - d = 0$

- linear
- sparse
- structured

PIPE EQUATIONS



$$p_2^2 - p_1^2 + k_1 q_1^{2.8359} = 0$$

where p_i **pressures**

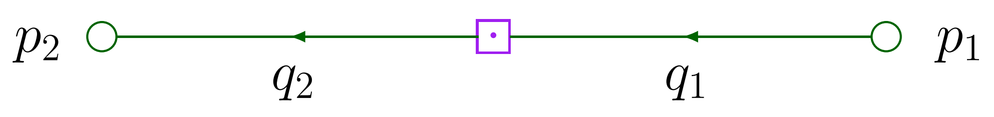
q_i **flows**

k_i **constants**

In general: $A^T p^2 + K q^{2.8359} = 0$

- non-linear
- sparse
- structured

COMPRESSOR CONSTRAINTS



$$q_1 - q_2 + z_1 \cdot c_1(p_1, q_1, p_2, q_2) \geq 0$$

where p_i **pressures**

q_i **flows**

z_i **0–1 variables**

$= 1$ if machine is on

c_i **nonlinear functions**

In general: $A_2^T q + z \cdot c(p, q) \geq 0$

- non-linear
- sparse
- structured
- 0–1 variables

OTHER CONSTRAINTS

Bounds on pressures and flows

$$\begin{aligned} p_{\min} \ &\leq \ p \ \leq \ p_{\max} \\ q_{\min} \ &\leq \ q \ \leq \ q_{\max} \end{aligned}$$

$\odot$ simple bounds on variables

OBJECTIVES

Many possible objectives

- $\odot$ maximize / minimize sum of pressures
- $\odot$ minimize compressor fuel costs
- $\odot$ minimize supply

+ combinations of these

STATISTICS

British Gas National Transmission System

- ⊙ 199 nodes
- ⊙ 196 pipes
- ⊙ 21 machines

Steady state problem

$\sim$400 variables

24-hour variable demand problem with 10 minute discretization

$\sim$58,000 variables

Challenge: Solve this in real time

TYPICAL PROBLEM

This problem is typical of real-world, large-scale applications

- ⊙ simple bounds
- ⊙ linear constraints
- ⊙ nonlinear constraints
- ⊙ structure
- ⊙ global solution “required”
- ⊙ integer variables
- ⊙ discretization

(SOME) OTHER APPLICATION AREAS

- minimum energy problems
- structural design problems
- traffic equilibrium models
- production scheduling problems
- portfolio selection
- parameter determination in financial markets
- hydro-electric power scheduling
- gas production models
- computer tomography (image reconstruction)
- efficient models of alternative energy sources

CLASSIFICATION OF OPTIMIZATION PROBLEMS