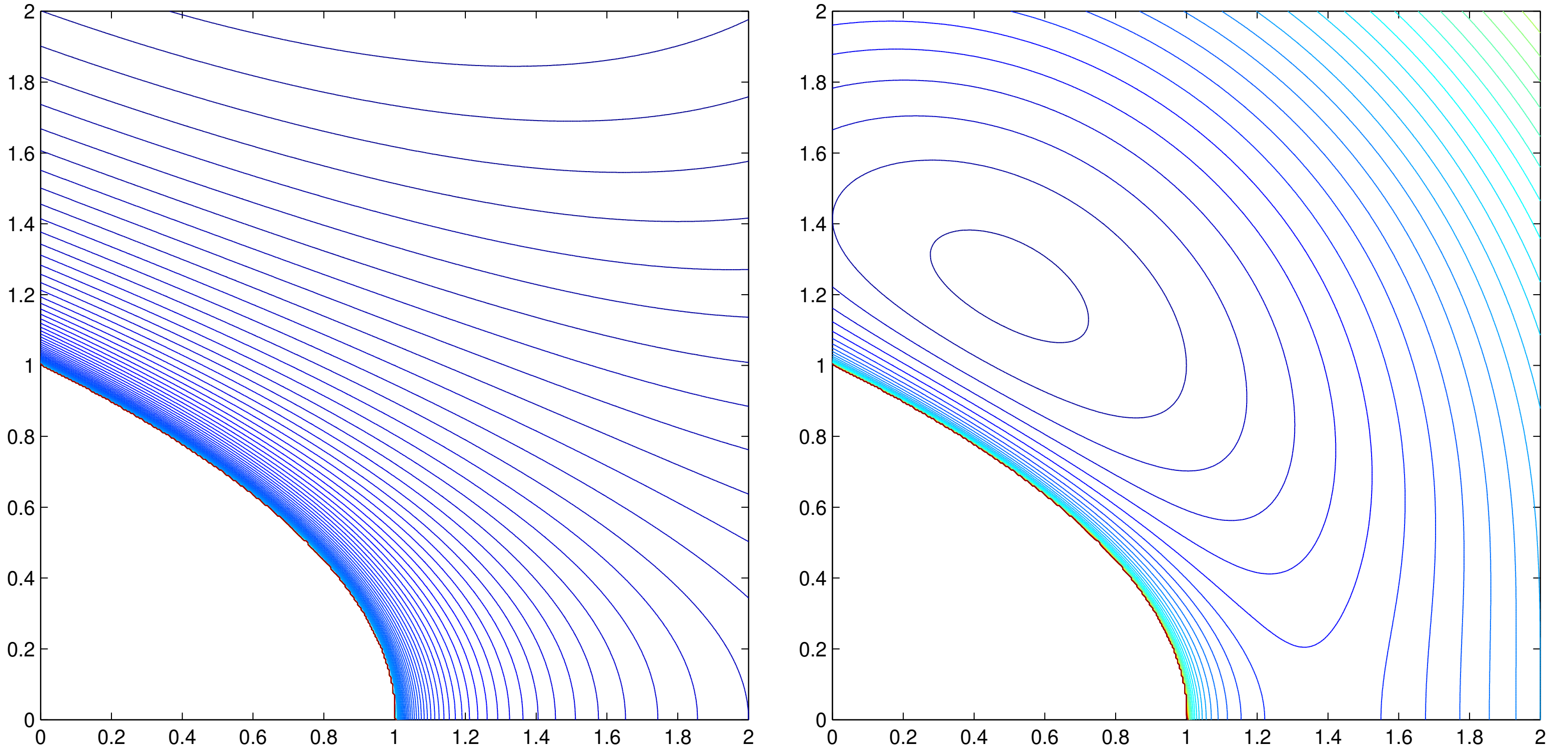
CONTOURS OF THE BARRIER FUNCTION



$\mu = 10$ $\mu = 1$

Barrier function for $\min x_1^2 + x_2^2$ subject to $x_1 + x_2^2 \geq 1$

FAST ASYMPTOTIC CONVERGENCE

Theorem 4.1. Suppose that f, $c \in \mathcal{C}^2$, that a subsequence $\{(x_k, y_k)\}$, $k \in \mathcal{K}$, of the practical primal-dual method converges to (x_*, y_*) satisfying second-order sufficiency conditions, that $A_{\mathcal{A}}(x_*)$ is full-rank, and that $(y_*)_{\mathcal{A}} > 0$. Then the starting point satisfies the inner-minimization termination test (i.e., $(x_k, y_k) = (x_k^{\mathrm{S}}, y_k^{\mathrm{S}})$) and the whole sequence $\{(x_k, y_k)\}$ converges to (x_*, y_*) at a superlinear rate (Q-factor 1.9998).

WHAT CAN WE DO IN THE NON-CONVEX CASE?

Solve the problem over a subspace

- instead of the B-conjugate subspace for CG, use the equivalent Lanczos orthogonal basis
  - Gram-Schmidt applied to CG (Krylov) basis $\mathcal{D}^i$
  - Subspace $\mathcal{Q}^i = \{s \mid s = Q^i s_q \text{ for some } s_q \in \mathrm{IR}^i\}$
  - Q^i is such that

$$Q^{i\,T} Q^i = I \text{ and } Q^{i\,T} B Q^i = T^i$$

    where T^i is tridiagonal and $Q^{i\,T} g = \|g\|_2\, e_1$
  - Q^i trivial to generate from CG $\mathcal{D}^i$