

## WHAT IS NONLINEAR PROGRAMMING?

**Nonlinear optimization**  $\equiv$  **nonlinear programming**

### Part 0: A gentle introduction to nonlinear optimization

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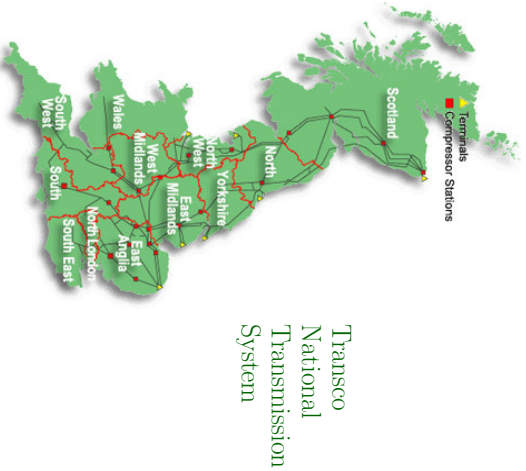
minimize  $f(x)$  subject to  $c_g(x) = 0$  and  $c_T(x) \geq 0$   
 $x \in \mathbb{R}^n$

MSc course on nonlinear optimization

## AN EXAMPLE

Optimization of a high-pressure gas network

British Gas (Transco)  
Oxford University  
RAL



minimize  $f(x)$  subject to  $c_g(x) = 0$  and  $c_T(x) \geq 0$   
where  $x$

**objective function**  $f : \mathbb{R}^n \rightarrow \mathbb{R}$

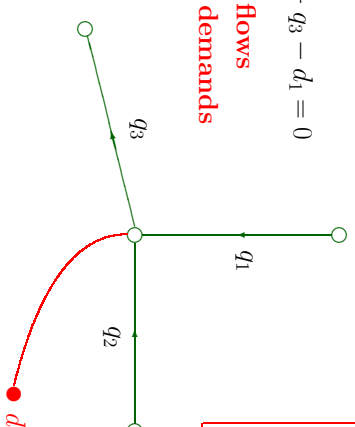
**constraints**  $c_g : \mathbb{R}^n \rightarrow \mathbb{R}^{m_g}$  ( $m_g \leq n$ ) and  $c_T : \mathbb{R}^n \rightarrow \mathbb{R}^{m_T}$

◦ there may also be integrality restrictions

## NODE EQUATIONS

$$q_1 + q_2 - q_3 - d_1 = 0$$

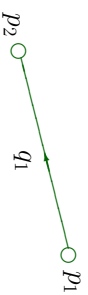
where  $q_i$  **flows**  
 $d_i$  **demands**



In general:  $Aq - d = 0$

- linear
- sparse
- structured

## PIPE EQUATIONS



$$p_2^2 - p_1^2 + k_1 q_1^2 s_{359} = 0$$

where  $p_i$ : **pressures**

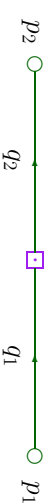
$q_i$ : **flows**

$k_i$ : **constants**

In general:  $A^T p^2 + K q^2 s_{359} = 0$

- non-linear
- sparse
- structured

## COMPRESSOR CONSTRAINTS



$$q_1 - q_2 + z_1 \cdot c_1(p_1, q_1, p_2, q_2) \geq 0$$

where  $p_i$ : **pressures**

$q_i$ : **flows**

$z_i$ : **0-1 variables**

= 1 if machine is on

$c_i$ : **nonlinear functions**

In general:  $A_2^T q + z \cdot c(p, q) \geq 0$

- non-linear
- sparse
- structured
- 0-1 variables

## OTHER CONSTRAINTS

### Bounds on pressures and flows

$$p_{\min} \leq p \leq p_{\max}$$

$$q_{\min} \leq q \leq q_{\max}$$

- simple bounds on variables

## OBJECTIVES

Many possible objectives

- maximize / minimize sum of pressures
- minimize compressor fuel costs
- minimize supply
- + combinations of these

## STATISTICS

British Gas National Transmission System

- 199 nodes
- 196 pipes
- 21 machines

Steady state problem

~400 variables

24-hour variable demand problem with 10 minute discretization

~58,000 variables

**Challenge:** Solve this in real time

## (SOME) OTHER APPLICATION AREAS

- minimum energy problems
- structural design problems
- traffic equilibrium models
- production scheduling problems
- portfolio selection
- parameter determination in financial markets
- hydro-electric power scheduling
- gas production models
- computer tomography (image reconstruction)
- efficient models of alternative energy sources

## TYPICAL PROBLEM

This problem is typical of real-world, large-scale applications

- simple bounds
- linear constraints
- nonlinear constraints
- structure
- global solution “required”
- integer variables
- discretization

## CLASSIFICATION OF OPTIMIZATION PROBLEMS

