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Fortran subroutines for handling sparse linear programming bases

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FORTRAN SUBROUTINES FOR HANDLING SPARSE LINEAR PROGRAMMING BASES

by

J.K. Reid

Abstract

In this report we present subroutines that implement a sparse variant (Reid, 1975) of the Bartels-Golub algorithm for handling linear programming bases. There are separate subroutines for factorization, solution of linear systems using such a factorization and updating the factorization to correspond to the replacement of one of the columns of the basis.

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1. **Introduction**

The purpose of this report is to present three subroutines that implement a sparse variant (Reid, 1975) of the Bartels-Golub algorithm. The subroutines are

a) **LA05A**, which produces a factorization

\[ A = LU \]  \hspace{1cm} (1.1)

of a given sparse nxn matrix \( A \), where \( L \) is a matrix whose inverse is held as the product

\[ L^{-1} = M_r M_{r-1} \ldots M_1 \]  \hspace{1cm} (1.2)

of \( r \) matrices \( M_i \) that differ from \( I \) in just one element, and \( U \) is a permutation of an upper triangular matrix

\[ \bar{U} = PUQ ; \]  \hspace{1cm} (1.3)

b) **LA05B**, which solves sets of equations

\[ Ax = b \]  \hspace{1cm} (1.4)

and

\[ A^T x = b \]  \hspace{1cm} (1.5)

using the factorization \((1.1)\); and

c) **LA05C** which revises the factorization \((1.1)\) when one of the columns of \( A \) is altered.

We expect the main application of these subroutines to be for solving linear programming problems (see for example Goldfarb and Reid, 1976) but they are also available as a basic tool for use in other optimization problems.
The algorithm is described by Reid (1975) and some test results on medium scale problems are also given by Reid (1975). Here we concentrate on describing the code itself. Although the reader may find it helpful to read the earlier paper, it is our intention that this report should be self-contained. In order to make the code more understandable we describe the algorithm slightly differently and use slightly different notation.

The subroutines presented here are in standard Fortran, and have been checked by the Bell Telephone Laboratories' Fortran verifier (Ryder, 1973), although those placed in the Harwell library are variants which use INTEGER*2 arrays. To aid readability of the code we avoid any backward jumps, all loops being programmed with the DO statement; this rule gives "well-structured" code without departing from standard Fortran. Occasionally we have needed a "DO WHILE" loop, for instance when looping along a linked list. In such cases we index the DO loop with an integer called IDUMMY (or JDUMMY, KDUMMY,...) to indicate that it is not used within the loop.

The code itself and its specification sheet are included in Section 6. So far as seems reasonable we have used comments in the code to explain its action. The underlying data structure is described in Section 2 and Sections 3, 4 and 5 describe the methods used in LA05A, LA05B and LA05C. Besides the code and specification sheets we include in Section 6 some comments on runs under a preprocessor which made execution counts available.

The author would like to thank I.S. Duff for reading a draft of this report and making some helpful suggestions regarding the presentation.
2. **Data structure**

In order to explain our choice of data structure we need to sketch the algorithm used. Further details of the algorithm are given in later sections. Immediately after the original factorization (LA05A) $L$ has a symmetric permutation

$$
\tilde{L} = PLP^{-1}
$$

(2.1)

which is a unit lower triangular matrix, but later (see next paragraph) it can only be said to be of the form (1.2). $U$ is always a permuted upper triangular matrix (see (1.3)).

If column $m$ of $A$ is changed to give the matrix $\tilde{A}$ then the factorization becomes

$$
\tilde{A} = LV
$$

(2.2)

where $V$ differs from $U$ only in column $m$, and this is $L^{-1}$ times column $m$ of $\tilde{A}$. $V$ may already be a permuted upper triangular matrix, in which case we take $U=V$, perhaps revising the permutation matrices $P$ and $Q$. If it is not, then further elementary row operations $M_{r+1}, \ldots, M_r$ are used to reduce it to such a form. This gives the new factorization

$$
\tilde{A} = L \bar{U}
$$

(2.3)

of the same form as previously, and associated permutation matrices $\bar{P}$ and $\bar{Q}$. Notice that $\bar{U}^{-1}$ differs from $L^{-1}$ only in the possible addition of more elementary row operation matrices $M_r, M_{r-1}, \ldots, M_1$ whereas $\bar{U}$ differs from $U$ in having a column replaced and elementary row operations applied.

The factorization (1.1) enables the sets of equations (1.4) and (1.5) to be written as

$$
x = U^{-1}(L^{-1}b) = U^{-1}(M_r \ldots M_1)b
$$

(2.4)

and

$$
x = L^{-T}(U^{-T}b) = M_r^T \ldots M_1^T(U^{-T}b)
$$

(2.5)
It follows that at no stage is access required to the matrices $M_i$ except in forward or reverse order and that updating need be nothing more complicated than adding additional matrices $M_i$. Therefore a sequential file which can be accessed forwards and backwards is a suitable storage mode and we have chosen to use the end of the arrays $A$ and $IND$. The number of matrices $M_i$ is held in $LENL$ of COMMON/LA05D/ and they themselves are held as the operators "add $A(K)$ times component $IND(K,1)$ of a given vector to component $IND(K,2)$", for $K = IA, IA-1, ..., IA-LENL+1$.

A more complicated storage mode is needed for $U$ because of the unpredictability of the extra non-zeros (fill-ins) that the row operations may produce. An early version (LA03) held each row of $U$ as a linked list. This enables fill-ins to be handled easily but

i) with INTEGER*2 links on the IBM370 we were limited to about 32000 non-zeros,

ii) accessing a row is likely to involve accessing well-separated items in the store, undesirable on a paged machine, and

iii) the column structure is not available so that replacing column $m$ requires a scan of the whole of $U$, for instance.

Because of these disadvantages we have decided to follow Gustavson (1972) in holding the rows of $U$ as a file of packed vectors and the pattern of the columns of $U$ as a file of lists of integers. The non-zeros of row $I$, say, are held in $A(K), A(K+1), ..., $ and corresponding column numbers are held in $IND(K,2), IND(K+1,2), ..., $ with $K$ held in $IP(I,1)$ and the number of elements (non-zeros) held in $IW(I,1)$. We use a completely separate file for the columns of $U$; here we do not store the numerical values but just the row numbers themselves. For column $J$ these are in $IND(K,1), IND(K+1,1), ..., $ where $K$ is in $IP(J,2)$ and the number of entries is in $IW(J,2)$. When a row operation is applied to $U$ we open a new entry for
the changed row and (temporarily) waste the space that the old row occupied. Any element in the row that changes to have modulus less than a given small number (SMALL of COMMON/LA05D/) is regarded as a zero and its entry in the column file is removed after having been interchanged with the last entry so that space at the end is released. Any fill-ins mean that an additional entry in the column structure is needed; this is placed at the end of the stored column if this is possible and if not a fresh entry is opened for the column and the old space is (temporarily) wasted. For both files there is a need for occasional "compression" to release wasted storage. This is done by subroutine LA05E, which is called to compress just one of the files whenever this is necessary. They are not treated together because usually the file of rows will need to be compressed more frequently since a new entry is made for every row operation applied to U. Spaces inside the file of rows are indicated by zero values of IND(K,2) and similarly zeros values of IND(K,1) indicate spaces in the column file. The overall length of the files, including internal spaces, are stored in LROW and LCOL of COMMON/LA05D/ and a count of the number of compresses (of either file) since last entry to LA05A is held in NCP of COMMON/LA05D/. We have found it convenient to hold the elements that are on the diagonal of the upper triangular matrix \( \tilde{U} = PUQ \) as first in their rows and columns; otherwise the entries are in arbitrary order within the rows and columns.

The actual compression in LA05E is done by scanning every element of the old file, including dummies, and moving just the genuine elements forward. Details of how the pointers are adjusted are given in comments. It is perhaps unfortunate that we have to look at all the dummies but the computer time taken here will be small compared with the time taken in other subroutines when the element is genuine, and this method leads to a
simple subroutine that does not require a sort of the pointers to starts of entries.

Finally in this section we describe how the permutations P and Q are held. \( IW(I,3) \) holds the row number in \( U \) of row \( I \) of \( \tilde{U} = PUQ \) and \( IW(J,4) \) holds the column number of column \( J \) of \( \tilde{U} \).

3. **Subroutine LA05A, which performs the original factorization**

The original factorization is performed using Gaussian elimination with the pivotal strategy of Markowitz (1957), but subject to the stability requirement that no pivot be less than \( u \) (a user-set parameter for which we have found the value 0.1 satisfactory) times the largest element in its row. Markowitz' strategy is to use a non-zero with least product of number of other non-zeros in its row and of other non-zeros in its column. To make the search for such an element fast we hold doubly linked lists of all rows having the same number of non-zeros and of all columns having the same number of non-zeros. This enables us to search columns of length 1, then rows of length 1 then columns of length 2, etc., stopping whenever we know that there cannot be a non-zero later in the sequence with better Markowitz cost than the best so far found. Normally this search will terminate very early. Once the pivot has been chosen we remove from their linked lists all the rows which have a non-zero in the pivot column and all the columns that have a non-zero in the pivot row because these may have their numbers of non-zeros changed.

Once the elimination is complete they are inserted in the list corresponding to their new numbers of non-zeros. We choose to search columns first to give a bias towards placing non-zeros in \( U \) rather than \( L \), because \( L \) can only grow in length whereas at a change of basis a column of \( U \) is removed; also the column coming into \( U \) at a change of basis has been produced from the corresponding column of \( A \) by operating with \( L^{-1} \), so is likely to have more non-zeros if \( L \) has more non-zeros.
The data structure used during elimination is similar to that used finally for U (see last section). As each elementary row operation is performed it is stored in its final position in the sequential file for L (using arrays A, IND from the end forwards). Initially A is stored in exactly the same way as U is eventually stored. During the elimination the row file holds all the rows but the column file contains only the submatrix of elements that have not been in a pivotal row or column (i.e. the "active" submatrix). Once the elimination is complete the column file for U is reconstructed from its row file.

For the user's convenience we require a different format for initial input of A. The non-zeros are passed in any order in the array A, with their row and column numbers in corresponding positions of arrays IND(·,1) and IND(·,2). The actual sorting of the non-zeros is done by subroutine MC20A, whose specification sheet is included in the appendix. It is fast since it handles each item that needs moving exactly three times. Although designed to order sparse matrices by columns we have been able to use it here by switching the roles of row and columns.

We do not permit any elements held in the row file to have the value zero because this leads to unnecessary work and use of storage. If any element has modulus less than SMALL (of COMMON/LA05D/ with default value zero) in the original matrix then it is removed before the row and column files are created. If any element with modulus less than SMALL is created during a row operation it is again removed. We recommend the user to reset SMALL to a positive value if he can since this will make underflows much less common and will save some storage.

The column file is used when choosing the pivot and in order to know which rows are active in the elimination itself. In neither case is there any need for the elements to be in order within their entry. We therefore make no attempt to order them, always adding any extra element to
the end of the entry and removing elements by overwriting the unwanted entry by the old last entry. It might be thought that the entries in the row file should be ordered so that during a row operation the two rows can be scanned in phase. We find it convenient to bring the elements of the pivot column to the front of both rows, but by using a full work vector of reals we can produce elegant and efficient code without the need for the remaining elements to be in any particular order. The full vector w is initialized to zero and is restored to zero after each use. For each row operation we

a) load the pivot row, excluding the pivot, into w.

b) scan the non-pivot row adding the required multiple of the appropriate component of w to each element and resetting the component of w to zero after its use.

c) scan the pivot row again to see which elements are still in w.

Each that is gives rise to a fill-in.

4. Subroutine LA05B, which solve sets of equations

Our data structure was chosen with one of its aims that the solution of sets of equations Ax=b and $A^T x = b$ should be straightforward and fast. It very often happens in linear programming applications that b is very sparse indeed and even x may have few non-zeros, and we want to exploit this feature. This leads to slightly more complicated code for solving $Ax=b$ so we describe the solution of $A^T x = b$ first.

Because the permutation $\tilde{U} = PUQ$ of U is an upper-triangular matrix, we may solve $U^T w = b$ by a forward substitution process. The presence of the permutations makes it convenient to use a work vector, so we first load the input vector b into w and set b to zero, then solve $U^T b = w$ by the forward substitution.
\[ b_{p_i} = (w_{q_i} - \sum_{j \neq p_i} u_{jq_i} b_{j})/u_{p_i q_i}, \quad i = 1, 2, \ldots, n. \quad (4.1) \]

As each non-zero \( b_{p_i} \) is calculated we run through row \( p_i \) of \( U \) (column \( p_i \) of \( U^T \)) performing the operation

\[ w_k = w_k - u_{p_i k} b_{p_i}, \quad k \neq q_i, \quad (4.2) \]

which can of course be skipped for zero \( b_{p_i} \). This ensures that at stage \( i \) of the process \( w_{q_i} \) has been updated fully and we find \( b_{p_i} \) by dividing by the pivot \( u_{p_i q_i} \). Since \( b \) has been initialized to zero we do not even have to look up the array containing \( p_i \) (\( p_i \) is held in I\(W\)(I,3)) if \( w_{q_i} \) is zero (\( q_i \) is held in I\(W\)(I,4)). Notice also how convenient it is to have the pivot \( u_{p_i q_i} \) stored at the row start. Once this process is complete we apply the sequence of elementary row operations that comprise \( L^{-T} \) to \( b \) so that it finally contains the required solution. Here we avoid null operations but for each operation we need an array look-up and to test for a zero vector component.

When solving \( Ax = b \) we begin by applying \( L^{-1} \) to \( b \) as a sequence of elementary row operations and again avoid null operations. Because of the permutations \( P \) and \( Q \) we next load \( b \) into \( w \) and set \( b \) to zero as in the last paragraph. Then we use the back-substitution

\[ b_{q_i} = (w_{p_i} - \sum_{j \neq q_i} u_{p_i j} b_{j})/u_{p_i q_i}, \quad i = n, n-1, \ldots, 1. \quad (4.3) \]

Unfortunately we cannot exploit zero components of \( b \) directly because \( U \) is held by rows but we do have the sparsity patterns of the columns of \( U \). Therefore after each non-zero component \( b_{q_i} \) is found we run through the pattern of column \( q_i \) of \( U \) marking those \( w_k \), other than \( w_{p_i} \), for which
$u_{kq_i} \neq 0$. At a later stage we know that $\sum_{j \neq q_i} u_{pq_i} b_j$ is zero if $w_{pq_i}$ is unmarked and therefore do not need to calculate this sum. The marking is actually performed by negating the pointers $IP(\cdot, l)$. Notice that $w$ is left unchanged as $L^{-1} b$. This is precisely the vector that is required by LA05C later if $b$ is to replace an existing column of the basis. It is not inefficient to use this as the only means of specifying the incoming column because in linear programming it always happens that $A^{-1} b$ is wanted before it is known which column is to leave the basis matrix $A$.

The great sparsity of the vectors that arise in typical linear programming applications led us to consider a condensed storage mode for them and we even wrote such a variant of LA05B. However we eventually decided against using it because the code is significantly more complicated and would execute faster only when the number of non-zeros in the output vector is less than about $\sqrt{n}$ because we have either to keep the non-zeros in order or perform a search whenever one is wanted. The loops of length $n$ that are avoided are simple and so likely to execute rapidly. We did find (see Section 6) that they were executed a great number of times, often giving the dominant execution count.

5. Subroutine LA05C, which updates the factorization

Subroutine LA05C updates the factorization following the replacement of column $m$ (MM in the code) of $A$ by $b$. We begin by removing column $m$ of $U$ and then inserting the vector $L^{-1} b$, calculated on a previous entry to LA05B, and stored in array $W$, as new column $m$ to make the matrix $V$ of equation (2.2). The permuted matrix

$$S = PVQ$$
is upper triangular except for one column (column M) which corresponds to the changed column and which we call the spike. The spike is read into our data structure in row order within S so that its last row number (in S) may be placed in the variable LAST. We refer to rows and columns M to LAST as "the bump". An example is shown in Figure 1 (page 15).

Our first aim is to alter the permutations in such a way that the length of the spike (size of the bump) is reduced. We begin by searching columns M+1, M+2, ..., LAST looking for a column (which we call a singleton column) having only one non-zero in the bump. If column J is such a column and we apply the symmetric permutation in which rows and columns M, M+1, ..., J-1 are all moved forward one place and row and column J becomes the new row and column M then the new matrix has exactly the same form as the old but now the spike is in column M+1 and is shorter by one component. In our example column 5 is such a column and the permuted matrix is shown in Figure 2. We can now look for a further singleton, apply another symmetric permutation and continue until none are found. It is more efficient, however, to find all the singletons then apply the composite symmetric permutation. This can be done by marking (by setting W(J)=1) column M and those of columns M+1, ..., LAST which are not singletons. We begin by marking column M itself and then marking all columns which have a non-zero in row M, since these certainly cannot be singletons. We then look at the marks for columns M+1, M+2, ... and for each marked column J we mark the columns having non-zeros in row J since again these cannot be singletons. In preparation for the next stage we also perform the symmetric permutation that places the spike at the end of the bump without altering the relative order of the other row and columns. The new order of columns is therefore

1) singletons, in unchanged order among themselves,
ii) non-singletons, in unchanged order among themselves and
iii) spike column. Our example has only one column singleton and its
form at this stage is shown in Figure 3. Because we hold pivots
(diagonal elements of the upper triangular matrix $U = PUQ$) first in their
row and column we do not need to store both $P$ and $Q$ explicitly and here we
are performing a symmetric permutation so pivots remain pivots. We
therefore revise only $P$ and use the storage for $Q$ as workspace.

These permutations leave us with a matrix $S = P_1VQ_1$ that is upper
triangular apart from row LAST (which we now call the spike row) and
this has non-zeros starting at column $M_1$, say, with $M_1 \geq M$. We refer
to rows and columns $M_1$ to LAST as "the bump", and apply an exactly
similar process to rows LAST-1, LAST-2,...,$M_1$ looking for rows that are
singletons in the bump as previously when looking for column singletons,
except that here we leave the spike where it is. This leaves us with
a spike in row LAST1, say, with LAST1 $\leq$ LAST, which commences in column $M_1$.
Our example (see Figure 3) begins with a row singleton in row 6
(row 7 of original matrix) and dealing with this makes row 4 into a
singleton. Dealing with both of these gives the matrix shown in
Figure 4.

We may be able to reduce the length of the spike still further if
column LAST1 of this new matrix $S_2 = P_2VQ_2$ is a singleton in the bump rows
$M_1$ to LAST1. Unless we have the trivial case $M_1$=LAST1 such a
singleton must have its non-zero in row LAST1-1 since otherwise this
would be a row singleton (and row singletons were all removed by the previous
set of permutations). We therefore perform the unsymmetric
permutation that makes row LAST1-1 become row $M_1$, column LAST1 become
column $M_1$ and the intervening rows and columns (rows $M_1+1,...,$LAST1-2 and
columns $M_1+1,...,$LAST1-1) all move forward one place. The new matrix
has the same structure but the spike now extends from column M1+1 of row \text{LAST1} (see Figure 5). We repeat the process (see Figure 6), continuing until the trivial case is reached or a non-singleton is discovered. Again we may delay the permutations and perform them together. Rows I of U that correspond to rows M1 to \text{LAST1} of S_2 are initially marked with W(I)=3, so we can test the last column for being a singleton. If it is, then we revise JM to point to the new last column and reset W(I) to 2, for the row being moved forward ready for the next singleton test. Similar action is taken as later singletons are found. The permutation P, stored in IW(\cdot,3) is revised when the process terminates, and M1 is increased to point to the new start of the spike row.

If these permutations have not reduced the matrix to upper triangular form (M1=LAST1), then the task is completed by application of a sequence of elementary Gaussian elimination steps. Each pivotal step begins with a search of the spike row for a non-zero in the pivot column. If one is found it is brought to the front of the spike. Next the pivot row is exchanged with the spike row if the stability test demands it (leading element of the pivot row less in modulus than \( u \) times that the other leading element) or if the sparseness suggests it (pivot row has greater number of non-zeros) while the stability test permits it (leading element of non-pivot row greater than \( u \) times leading element of pivot row). The actual row elimination is performed by code identical with that of LA05A.

Finally the permutation \( \tilde{Q} \) is constructed from \( \tilde{P} \), now stored in IW(\cdot,3), by using the fact that pivotal elements are always first in the stored rows.
Figure 1. Original matrix

Figure 2. After column singleton moved to front of bump

Figure 3. After spike moved to end of bump

Figure 4. After treating row singletons
Figures 5 and 6. After treating first and second singleton spike column

6. Code and specification sheets

Given in this section is the code itself and its specification sheet. Also given is the specification sheet of the Harwell subroutine MC20A which is called by LA05A to sort the non-zeros by columns.

The code given is the single-length standard Fortran version. It contains comments which allow the Harwell subroutine OE04 to convert to double-length standard Fortran (version labelled D), double-length IBM Fortran (version I) or single-length IBM Fortran (version J). These comments consist of statements labelled with the version to which they apply. Each is an alternative to the Fortran statement that immediately follows it.

We have run some linear programming problems, including most of those by Goldfarb and Reid (1975), after treating the code by a preprocessor (Harwell subroutine OE02) that inserts additional statements for counting the number of executions of each statement. We used the steepest edge simplex algorithm, as described by Goldfarb and Reid (1975). The highest counts were invariably in LA05B. The simple
loops of length N (i.e. DO 113, DO 140, DO 303, DO 315) were always very prominent and particularly so in the sparser cases. That it is worthwhile to test for zero components in the vectors being operated upon, even in the relatively complicated case of applying $U^{-1}$, was amply demonstrated. In LA05C, rather surprising, the heaviest count was usually in the beginning of the loop DO 110, that is the simple search for non-zeros in the incoming column. Another high count in LA05C, but small compared with those of LA05B, was often in the simple loop DO 270 that sets the vector w to zero prior to its use for a sequence of eliminations. The success of the singleton selection process was indicated by the much higher counts obtained in the first scan of the bump (looking for singleton columns) than in later scans (looking for singleton rows and dealing with singleton spikes). All the LA05A counts were much lower than the high counts of LA05B and LA05C. If execution speed is very important it might be worthwhile to machine code LA05B, or parts of it, and the loop that finds the non-zeros in the incoming column for LA05C.
1. **Purpose**

This package of subroutines will factorize a matrix, solve corresponding systems of linear equations and update the factorization when a column of the matrix is altered, exploiting sparsity in all cases. Its primary application is likely to be for handling linear programming bases. It has three entries:

(a) LA05A factorizes a given matrix A.

(b) LA05B subsequently calculates \( A^{-1}b \) or \( A^{-T}b \) for a given vector b, using the factorized A.

(c) LA05C modifies the factorization to correspond with the replacement of a column of the matrix by the vector b of a previous LA05B entry which calculated \( A^{-1}b \).

2. **Argument Lists**

CALL LA05A(A,IND,NZ,IA,N,IP,IW,W,G,U)
CALL LA05B(A,IND, IA,N,IP,IW,W,G,B,TRANS)
CALL LA05C(A,IND, IA,N,IP,IW,W,G,U,M)

A is a REAL(DOUBLE PRECISION for the D version) array of length IA. On entry to LA05A it must be set to contain, in any order, the non-zeros of A. On exit from LA05A and on entry to and exit from LA05B/C it contains the factors of the current matrix A. It is altered by LA05A and C, and must not be altered by the user except prior to an LA05A entry.

IND is an INTEGER(INTEGER*2 for IBM versions) array of dimensions (IA,2). On entry to LA05A IND(K,1),IND(K,2) must be set to contain the row and column number of the non-zero held in A(K) for K=1,2,...NZ. It is altered by LA05A and C, and must not be altered by the user except prior to an LA05A entry.

NZ (INTEGER) must be set by the user to the number of non-zeros in A. It is used by LA05A only and is not altered by it.

IA (INTEGER) must be set by the user to indicate the size of arrays A and IND. Advice on the choice of the size is given in §3. It is not altered by LA05A/B/C.

N (INTEGER) must be set by the user to the order of A. For the IBM versions it may not exceed 32767 because of the use of INTEGER*2 arrays. It is not altered by LA05A/B/C.

IP is an INTEGER work array of length N*2. It must not be altered by the user except prior to an LA05A entry.
IW is an INTEGER(INTEGER*2 for IBM versions) work array of length 
N*2 the first half of which must not be altered by the user except 
prior to an LA05A entry. The second half is not used by LA05B/C.

W is a REAL (or DOUBLE PRECISION in the D version) working array of 
length at least N. It is used to transmit information about an 
incoming column between an LA05B entry with TRANS=.FALSE. and a 
subsequent LA05C entry, and therefore should not be altered between 
two such entries.

G (REAL or DOUBLE PRECISION in the D version) is used to output 
information about the stability of the factorization and error 
conditions. After a successful entry G is positive and equal to 
the modulus of the largest element in any of the reduced matrices. 
This is explained further in section 6. After an unsuccessful 
entry it is set negative (see §4 for details).

U (REAL or DOUBLE PRECISION in the D version) is a number set by the 
user in the range 0<U≤1 to control the choice of pivots; if 
U=1 it is reset to 1 and if U≤0 it is reset to the relative 
floating-point accuracy. When searching for a pivot any element 
less than U times the largest element in its row is excluded. Thus 
decreasing U biases the algorithm towards maintaining sparsity at 
the expense of stability and vice-versa. The value 0.1 has been 
found satisfactory in test examples. It is used only by LA05A and 
LA05C.

B is a REAL (or DOUBLE PRECISION in the D version) array of length N 
used by LA05B to input b and output A⁻¹b (TRANS=.FALSE.) or A⁻¹b 
(TRANS=.TRUE.). It is used by LA05B only.

TRANS is a LOGICAL variable which must be set to .FALSE. if A⁻¹b is 
required from LA05B and to .TRUE. if A⁻¹b is wanted. It is used only 
by LA05B and is not altered by it.

M (INTEGER) is the column number in A of the column to be replaced 
in an LA05C entry. It is used by LA05C only and is not altered 
by it.

3. Storage considerations

The matrix is factorized into a product LU. The matrix L is 
stored as a product of matrices Lᵢ differing from the unit matrix I on 
only one element. Each matrix Lᵢ is stored in one position of A and 
IND and LENL in COMMON (see §5) holds the number of such matrices. U 
is a permutation of an upper triangular matrix and its number of 
non-zeros is held in LENU of COMMON (see §5). It is held in A and IND 
as a file containing separate ordered lists for each row and column. 
This file will need occasional compression to release space used by 
 altered rows and columns. This compression (actually performed by the 
subroutine LA05E) will not add a significant overhead to the computational 
cost if it happens less often than, say, alternate calls of LA05C. If 
it is required more than twenty times in a single call of LA05C then this 
call is aborted and a diagnostic is printed. If such a call follows 
a long sequence of LA05C calls then it can probably be corrected by a fresh
LA05A call. Both L and U are held in A and IND so IA (the size of A and IND) must exceed LLEN+LENU (the number of non-zeros in L and U) by a margin sufficient to avoid overfrequent compression of the file holding U. The adequacy of the length IA may be judged by monitoring NCP (in COMMON, see §5) which accumulates the number of times the U file is compressed since the last entry to LA05A.

4. **Error diagnostics**

After an unsuccessful entry a message is output on the line printer (unless suppressed or switched to another stream, see §5) and G is set negative to indicate one of the following conditions.

-1 'N is not positive'.
-2 'Row (or col) j has no elements'.
-3 'Element k is in row i and column j' (one of which is out of range).
-4 'There is more than one entry in row i and column j'.
-5 'The matrix was found singular in pivotal step k. Row (or col) j is dependent on rows (or cols) k,l,...'.
-6 'Singular matrix created by replacement of col m'.
-7 'IA is too small'.

Diagnostics 1 to 5 may result from an entry into LA05A and 6 or 7 may result from an LA05C entry. Also error returns (with G unchanged) may result from LA05B or LA05C if G is negative, indicating a previous error return.

5. **Use of Common**

The subroutines contain the following common block

```plaintext
COMMON/LA05D/SMLL,LP, LLEN,LENU,NCP,LROW,LCOL
```

(called LA05DD in the D version).

SMALL is a REAL variable (DOUBLE PRECISION in the D version) given the default value zero by BLOCK DATA. Its purpose is explained in §6.

LP is an INTEGER variable, given the default value 6 by BLOCK DATA, and used for stream number for diagnostic messages. Messages are suppressed if LP=0.

LENL,LENU,NCP are INTEGER variables giving information about use of the store (see §3).
LROW and LCOL are INTEGER variables used internally by the LA05 subroutines to hold the lengths of the files holding U by rows and its structure by columns.

If the user includes a common statement of the above form in his program then he may alter SMALL and LP from their default values and he may inspect LENL, LENU and NCP.

None of the variables LENL,...,LCOL may be altered by the user except prior to an LA05A entry.

6. Method and general notes

LA05A decomposes A into triangular factors using sparse matrix techniques similar to those of MA18A, documented in AERE Report R.6844.

Changing a column of A corresponds to changing a column of the upper triangular factor so that it is no longer a permutation of a triangular matrix and further row operations and/or permutations are needed to restore it to this form.

To control stability all pivots are chosen so that the multiples of a row that are added to another are always less than 1/U and stability is monitored by the parameter G, which is set to the modulus of the largest element in A or any of the upper triangular matrices to which it is reduced. If \( \varepsilon \) is the relative accuracy of the computation in use then the solutions obtained will have errors comparable with those of a perturbed system with matrix \( A + \delta A \), elements of \( \delta A \) being less than a small multiple of \( \varepsilon G \). Any elements of the upper triangular factor that are less than SMALL (of COMMON, see §4) are reset to zero; this has an effect comparable with that of making a perturbation to A whose elements have size about SMALL. We recommend the user to reset this to a positive value if he can, because this will save most underflow interrupts and some storage.

An LA05A call is normally followed by a long sequence of calls of LA05B and C. The time taken by LA05B will grow steadily as the number of non-zeros in the factors of A grows and eventually it will be more economic to call LA05A with the current matrix A and continue from this. A further call of LA05A may also be needed because of instability; large values of G are an indication of trouble but a better test is to calculate

\[
t = Ax - b \quad \text{or} \quad A^T x = b
\]

where \( x \) is the result of a LA05B call and compare \( r_i \) with \( \sum a_{ij}x_j \) (or \( \sum a_{ji}x_j \)).

7. Other subroutines

This is in fact a package of subroutines whose names are LA05A, LA05B, LA05C and LA05E.

December, 1975
1. **Purpose**

   a) **MC20A**: To sort the non-zeros of a sparse matrix from arbitrary order to column order, unordered within each column.

   b) **MC20B**: To sort the non-zeros within each column of a sparse matrix stored by columns.

2. **Argument lists**

   ```
   CALL MC20A(NC,MAXA,A,INUM,JPTR,JNUM,JDISP)
   CALL MC20B(NC,MAXA,A,INUM,JPTR)
   ```

   **NC** (INTEGER) must be set by the user to the number of matrix columns, and for the IBM versions it must not exceed 32767+JDISP. It is not altered by MC20A/B.

   **MAXA** (INTEGER) must be set by the user to the number of matrix non-zeros. It is not altered by MC20A/B.

   **A** is a REAL (DOUBLE PRECISION in D version) array of length MAXA. For entry to MC20A the user must set it to contain the non-zeros in any order. On exit from MC20A they are reordered so that column 1 precedes column 2 which precedes column 3, etc., but the order within columns is arbitrary. This format is required for entry to MC20B. On exit from MC20B the non-zeros are also ordered within each column.

   **INUM** is an INTEGER(INTEGER*2 for IBM versions) array of length MAXA. On entry to and exit from MC20A/B the absolute value of INUM(K) is the row number of the element in A(K). The values, including signs, are moved so the user is at liberty to use these signs as flags attached to the non-zeros.

   **JPTR** is an INTEGER array of length NC. It is not required to be set for entry to MC20A. On exit from MC20A and on entry to and exit from MC20B it contains the position in A of the first element of column J, J=1,2,...,NC.

   **JNUM** is an INTEGER(INTEGER*2 for IBM versions) array of length MAXA. On entry to MC20A JNUM(K)+JDISP is the column number of the element held in A(K). It is destroyed by MC20A.

   **JDISP** (INTEGER) must be set by the user to his required displacement for column numbers, in the range [0,32767]. Normally zero will be suitable, but positive values permit matrices with up to 65534 columns to be handled by the IBM versions for which JNUM is an INTEGER*2 array. JDISP is not altered by MC20A.
3. Notes

It is expected that this subroutine will be called by other library subroutines but not by the user directly. There are no checks on the validity of the data and no error exits.

4. Method

MC20A is an in-place sort algorithm which handles each item to be sorted exactly 3 times, so it is of order $N^2$. The number of elements in each column is first obtained by a counting pass. The space needed by each column is allocated. Each element in turn is made the "current element" and examined to see if it is in place. If not, it is put into the next location allotted for the column it occurs in, and the element displaced makes the current element. This chain of displacing elements continues until the first element examined in the chain is located and stored. Then the next item is examined. A flag prevents an element being moved twice.

MC20B is a pairwise interchange algorithm of maximum order $r(r-1)/2$, for each column, where $r$ is the number of elements in the column.

November, 1975
C I AND J ARE IBM FORTRAN DOUBLE AND SINGLE LENGTH VERSIONS
C D AND S ARE STANDARD FORTRAN DOUBLE AND SINGLE LENGTH VERSIONS
C
SUBROUTINE LA0SAD(A,IND,NZ,IA,N,IP,IW,W,G,U)
SUBROUTINE LA0SA (A,IND,NZ,IA,N,IP,IVW,W,G,U)
INTEGER IP(N,2),RC(2,3)
C
INTEGER*2 IND(IA,2),IW(N,B)
INTEGER IND(IA,2),IW(N,B)
C
DOUBLE PRECISION A(IA),AMAX,AU,AM,U.SMALL,W(N)
REAL A(IA),AMAX,AU,AM,U.SMALL,W(N)
C
COMMON/LA05DD/SMALL,LP,LENL,LENU,NCP,LROW,LCOL
C
IP(I,1),IP(I,2) POINT TO THE START OF ROW/COL I.
C
IW(I,1),IW(I,2) HOLD THE NUMBER OF NON-ZEROS IN ROW/COL I.
C
DURING THE MAIN BODY OF THIS SUBROUTINE THE VECTORS IW(*,3),IW(*,5),
C
IW(*,7) ARE USED TO HOLD DOUBLY LINKED LISTS OF ROWS THAT HAVE
C
NOT BEEN PIVOTAL AND HAVE EQUAL NUMBERS OF NON-ZEROS.
C
IW(*,4),IW(*,6),IW(*,8) HOLD SIMILAR LISTS FOR THE COLUMNS.
C
IW(I,3),IW(I,4) HOLD FIRST ROW/COLUMN TO HAVE I NON-ZEROS
C
OR ZERO IF THERE ARE NONE.
C
IW(I,5),IW(I,6) HOLD ROW/COL NUMBER OF ROW/COL PRIOR TO ROW/COL I
C
IN ITS LIST, OR ZERO IF NONE.
C
IW(I,7),IW(I,8) HOLD ROW/COL NUMBER OF ROW/COL AFTER ROW/COL I
C
IN ITS LIST, OR ZERO IF NONE.
C
FOR ROWS/COLS THAT HAVE BEEN PIVOTAL IW(I,5),IW(I,6) HOLD NEGATION OF
C
POSITION OF ROW/COL I IN THE PIVOTAL ORDERING.
C
COMMON/LA05D / SMALL,LP,LENL,LENU,NCP,LROW,LCOL
DATA RC(1,1),RC(1,2),RC(1,3),RC(2,1),RC(2,2),RC(2,3)
1 / 1H,1HO,1HW,1HC,1HO,1HL/
C
DATA EPS/2.3E-16/
C
EPS IS THE RELATIVE ACCURACY OF FLOATING-POINT COMPUTATION
C
DATA EPS/1.0E-6 /
IF(U,GT.1.)U=1.
IF(U,LT.EPS)U=EPS
IF(N,LT.1)GO TO 520
G=0.
DO 5 I=1,N
W(I)=0.
DO 5 J=1,N
5 IW(I,J)=0
C
C FLUSH OUT SMALL ENTRIES. CCOUNT ELEMENTS IN ROWS AND COLUMNS
L=1
LENU=NZ
DO 20 IDUMMY=1,NZ
IF(L.GT.LENU)GO TO 25
DO 10 K=L,LENU
C IF(DABS(A(K)).LE.SMALL)GO TO 15
IF(ABS(A(K)).LE.SMALL)GO TO 15
I=IND(K+1)
J=IND(K+2)
C G=DMAX1(DABS(A(K)).G)
G=AMAX1(ABS(A(K)).G)
IF(I.LT.1.OR. I.GT.N)GO TO 540
IF(J.LT.1.OR. J.GT.N)GO TO 540
IW(I+1)=IW(I+1)+1
IW(J+2)=IW(J+2)+1
GO TO 25
15 L=K
A(L)=A(LENU)
IND(L+1)=IND(LENU+1)
IND(L+2)=IND(LENU+2)
20 LENU=LENU-1
C
25 LENL=0
LROW=LENU
LCOL=LROW
C MCP IS THE MAXIMUM NUMBER OF COMPresses PERMITTED BEFORE AN
C ERROR RETURN RESULTS.
C MCP=MAX0(N/10,20)
NCP=0
C CHECK FOR NULL ROW OR COLUMN AND INITIALIZE IP(I,2) TO POINT
C JUST BEYOND WHERE THE LAST COMPONENT OF COLUMN I OF A WILL
C BE STORED.
K=1
DO 28 IR=1,N
K=K+IW(IR)+2
IP(IR+2)=K
DO 28 L=1,2
IF(IW(IR+L).LE.0)GO TO 580
28 CONTINUE
C REORDER BY ROWS
C CALL MC20AD(N,LENU,A,IND(1,2),IP,IND(1,1),0)
C CHECK FOR DOUBLE ENTRIES WHILE USING THE NEWLY CONSTRUCTED
C ROW FILE TO CONSTRUCT THE COLUMN FILE. NOTE THAT BY PUTTING
C THE ENTRIES IN BACKWARDS AND DECREASING IP(J,2) EACH TIME IT
C IS USED WE AUTOMATICALLY LEAVE IT POINTING TO THE FIRST ELEMENT.
CALL MC2OA (N,LENU,A,IND(1,2),IP,IND(1,1),0)
KL=LENU
DO 40 II=1,N
IR=N-1-II
KP=IP(IR,1)
DO 30 K=KP,KL
J=IND(K,2)
IF(IW(J,5),EQ.,IR)GO TO 500
IW(J,5)=IR
KR=IP(J,2)-1
IP(J,2)=KR
30 IND(KR,1)=IR
40 KL=KP-1
C
C SET UP LINKED LISTS OF ROWS AND COLS WITH EQUAL NUMBERS OF NON-ZEROS.
DO 100 L=1,N
DO 100 I=1,N
NZ=IW(I,L)
IN=IW(NZ,L+2)
IW(NZ,L+2)=I
IW(I,L+6)=IN
IW(I,L+4)=0
100 IF(IN,NE.,0)IW(IN,L+4)=I
C
C
C START OF MAIN ELIMINATION LOOP.
DO 480 IPV=1,N
C FIND PIVOT. JCOST IS MARKOWITZ COST OF CHEAPEST PIVOT FOUND SO FAR.
C WHICH IS IN ROW IPP AND COLUMN JP.
JCOST=N*N
C LOOP ON LENGTH OF COLUMN TO BE SEARCHED
DO 155 NZ=1,N
IF(JCOST.LE.(NZ-1)**2)GO TO 183
J=IW(NZ,4)
155

C SEARCH COLUMNS WITH NZ NON-ZEROS.
DO 131 IDUMMY=1,N
IF(J,LE.,0)GO TO 133
KP=IP(J,2)
IF(KCOST.GE.JCOST)GO TO 150
JCOST=KCOST
IPP=I
JP=J
IF(JCOST.LE.(NZ-1)**2)GO TO 183
150 CONTINUE
151 I=IW(I+7)
155 CONTINUE
C
C PIVOT FOUND.
C REMOVE ROWS AND COLUMNS INVOLVED IN ELIMINATION FROM ORDERING VECTORS.
183 KP=IP(JP+2)
KL=IW(JP+2)+KP-1
DO 195 L=1,2
DO 190 K=KP,KL
I=IND(K.L)
IL=IW(I.L+4)
IN=IW(I.L+6)
IF(IL.EQ.0)GO TO 185
IW(IL.L+6)=IN
GO TO 190
185 NZ=IW(I.L)
IW(NZ.L+2)=IN
190 IF(IN.GT.0)IW(IN.L+4)=IL
KP=IP(IPP+1)
195 KL=KP+IW(IPP+1)-1
C STORE PIVOT
IW(IPP+5)=-IPV
IW(JP+6)=-IPV
C ELIMINATE PIVOTAL ROW FROM COLUMN FILE AND FIND PIVOT IN ROW FILE.
DO 219 K=KP,KL
J=IND(K.L)
KPC=IP(J.L)
IW(J,L)=IW(J,L)-1
KLC=KPC+IW(J,L)
DO 215 KC=KPC,KLC
IF(IPP.EQ.IND(KC,L))GO TO 216
215 CONTINUE
216 IND(KC,L)=IND(KLC,L)
IND(KLC,L)=0
219 IF(J.EQ.JP)KR=K
C BRING PIVOT TO FRONT OF PIVOTAL ROW.
AU=A(KR)
A(KR)=A(KP)
A(KP)=AU
IND(KR+2)=IND(KP+2)
IND(KP+2)=JP

C PERFORM ELIMINATION ITSELF. LOOPTING ON NON-ZEROS IN PIVOT COLUMN.
N28=I(W(JP+2))
IF(N28.EQ.0) GO TO 468
DO 467 NC=1,N28
KC=I(P(JP+2)+NC-1
IR=IND(KC+1)

C SEARCH NON-PIVOT ROW FOR ELEMENT TO BE ELIMINATED.
KR=I(P(IR+1)
KRL=KR+I(W(IR+1))-1
DO 290 KNP=KR,KRL
IF(JP.EQ.IND(KNP+2)) GO TO 300
290 CONTINUE

C BRING ELEMENT TO BE ELIMINATED TO FRONT OF ITS ROW.
300 AM=A(KNP)
A(KNP)=A(KR)
A(KR)=AM
IND(KNP+2)=IND(KR+2)
IND(KR+2)=JP
AM=-A(KR)/A(KP)

C COMPRESS ROW FILE UNLESS IT IS CERTAIN THAT THERE IS ROOM FOR NEW ROW.
IF(ROW+I(W(IR+1))+I(W(IPP+1))+LENL.LE.IA) GO TO 340
IF(NCP.GE.MCP OR LENU+I(W(IR+1))+I(W(IPP+1))+LENL.GT.IA) GO TO 600

C CALL LA05ED(A,IND(1,2),IP,N,IW,IA **TRUE**) DI/
CALL LA05E(A,IND(1,2),IP,N,IW,IA **TRUE**)
KP=I(P(IPP+1)
KR=I(P(IR+1)
340 KRL=KR+I(W(IR+1))-1
KQ=KP+1
KPL=KP+I(W(IPP+1))-1

C PLACE PIVOT ROW (EXCLUDING PIVOT ITSELF) IN W.
 IF(KQ.GT.KPL) GO TO 350
 DO 345 K=KQ,KPL
 J=IND(K,2)
345 W(J)=A(K)
350 IP(IR+1)=LROW+1
C
C TRANSFER MODIFIED ELEMENTS.
IND(KR+2)=0
KR=KP+1
IF(KR.GT.KRL)GO TO 380
DO 370 KS=KR,KRL
J=IND(KS+2)
AU=A(KS)+AM*W(J)
IND(KS+2)=0
C IF ELEMENT IS VERY SMALL REMOVE IT FROM U.
C IF(DABS(AU).LE.SMALL)GC TO 365
IF(ABS(AU).LE.SMALL)GO TO 365
C
G=DMAX1(G,DABS(AU))
G=AMAX1(G,ABS(AU))
LROW=LROW+1
A(LROW)=AU
IND(LROW+2)=J
GO TO 370
365 LENU=LENU-1
C REMOVE ELEMENT FROM COL FILE.
K=IP(J+2)
KL=K+IW(J+2)-1
IW(J+2)=KL-K
DO 366 KK=KL,KL
IF(IND(KK+1).EQ.IR)GO TO 367
366 CONTINUE
367 IND(KK+1)=IND(KL+1)
IND(KL+1)=0
370 W(J)=0.
C
C SCAN PIVOT ROW FOR FILLS.
380 IF(KQ.GT.KPL)GO TO 435
DO 430 KS=KQ,KPL
J=IND(KS+2)
AU=AM*W(J)
C IF(DABS(AU).LE.SMALL)GO TO 430
IF(ABS(AU).LE.SMALL)GO TO 430
LROW=LROW+1
A(LROW)=AU
IND(LROW+2)=J
LENU=LENU+1
C
C CREATE FILL IN COLUMN FILE.
NZ=IW(J+2)
K=IP(J+2)
KL=K+NZ-1
C IF POSSIBLE PLACE NEW ELEMENT AT END OF PRESENT ENTRY.
IF(KL,NE.,LCOL)GO TO 390
IF(LCOL+LENL,GE.,IA)GO TO 400
LCOL=LCOL+1
GO TO 395
390 IF(IND(KL+1,1).NE.0)GO TO 400
395 IND(KL+1,1)=IR
GO TO 425
C NEW ENTRY HAS TO BE CREATED.
400 IF(LCOL+LENL+NZ,1,LT.,IA)GO TO 410
C COMPRESS COLUMN FILE IF THERE IS NOT ROOM FOR NEW ENTRY.
IF(NCP,GE.,MCP, .OR., LENU+LENL+NZ,1,GE.,IA)GO TO 600
C CALL LA05ED(A,IND,IP(1,2),N,IW(1,2),IA ) .FALSE.)
CALL LA05E(A,IND,IP(1,2),N,IW(1,2),IA ) .FALSE.)
K=IP(J+2)
KL=K+NZ-1
C TRANSFER OLD ENTRY INTO NEW.
410 IP(J+2)=LCOL+1
DO 420 KK=K+KL
LCOL=LCOL+1
IND(LCOL+1)=IND(KK+1)
420 IND(KK+1)=0
C ADD NEW ELEMENT.
LCOL=LCOL+1
IND(LCOL+1)=IR
C425 G=DMAX1(G,DABS(AU))
425 G=AMAX1(G, ABS(AU))
IW(J+2)=NZ+1
430 W(J)=0.
435 IW(IR+1)=LROW+1-IP(IR+1)
C
C STORE MULTIPLIER
IF(LENL+LCOL+1,LE.,IA)GO TO 450
C COMPRESS COL FILE IF NECESSARY.
IF(NCP,GE.,MCP)GO TO 600
CALL LA05ED(A,IND,IP(1:2),N,IW(1:2),IA **FALSE.**)
CALL LA05E(A,IND,IP(1:2),N,IW(1:2),IA **FALSE.**)
450 K=IA-LENL
LENL=LENL+1
A(K)=AM
IND(K,1)=IPP
IND(K,2)=IR
LENU=LENU-1
467 CONTINUE
C
C INSERT ROWS AND COLUMNS INVOLVED IN ELIMINATION IN LINKED LISTS
C OF EQUAL NUMBERS OF NON-ZEROS.
468 K1=IP(IP,2)
   K2=IW(IP,2)+K1-1
   IW(IP,2)=0
   DO 480 L=1,2
   IF(K2.LT.K1)GO TO 475
   DO 470 K=K1,K2
   IR=IND(K,L)
   IF(L.EQ.1)IND(K,L)=0
   NZ=IW(IR,L)
   IF(NZ.LE.0)GO TO 630
   IN=IW(NZ,L+2)
   IW(IR,L+6)=IN
   IW(IR,L+4)=0
   IW(NZ,L+2)=IR
   470 IF(IN.NE.0)IW(IN,L+4)=IR
475 K1=IP(IPP,1)+1
480 K2=IW(IPP,1)+K1-2
C
C RESET COLUMN FILE TO REFER TO U AND STORE ROW/COL NUMBERS IN
C PIVOTAL ORDER IN IW(*,3),IW(*,4)
   DO 482 I=1,N
   J=-IW(I,5)
   IW(J,3)=I
   J=-IW(I,6)
   IW(J,4)=I
   482 IW(I,2)=0
   DO 485 I=1,N
   KP=IP(I,1)
   KL=IW(I,1)+KP-1
DO 485 K=KP*KL
   J=IND(K+2)
   485  IW(J+2)=IW(J+2)+1
   K=1
   DO 487 I=1,N
   K=K+IW(I+2)
   487  IP(I+2)=K
   LCDL=K-1
   DO 490 II=1,N
   I=IW(II+3)
   KP=IP(I+1)
   KL=IW(I+1)+KP-1
   DO 490 K=KP,KL
   J=IND(K+2)
   KN=IP(J+2)-1
   490  IP(J+2)=KN
   IND(KN+1)=I
   GO TO 720
C
THE FOLLOWING INSTRUCTIONS IMPLEMENT THE FAILURE EXITS.
C
500 IF(LP.GT.0)WRITE(LP,510)IR,J
510 FORMAT(//34X,35HTHERE IS MORE THAN ONE ENTRY IN ROW.I5,
11H AND COLUMN.I5)
   G=4.
   GO TO 700
520 IF(LP.GT.0)WRITE(LP,530)
530 FORMAT(//34X,17HN IS NOT POSITIVE)
   G=1.
   GO TO 700
540 IF(LP.GT.0)WRITE(LP,550)K,I,J
550 FORMAT(//34X,7THE ELEMENT.I7+10H IS IN ROW.I5,11H AND COLUMN.I5)
   G=-3.
   GO TO 700
580 IF(LP.GT.0)WRITE(LP,590)(RC(L+I),I=1,3),IR
590 FORMAT(//34X,3A1,I5,16H HAS NO ELEMENTS)
   G=-2.
   GO TO 700
600 IF(LP.GT.0)WRITE(LP,610)
610 FORMAT(//34X,15HIA IS TOO SMALL)
   G=-7.
   GO TO 700
630 IPV=IPV+1
   IW(IPV+1)=IR
DO 640 I=1,N
   II=-IW(I*L+4)
640 IF(II.GT.0)IW(I*I+1)=1
   IF(LP.GT.0)WRITE(LP,650)(RC(L*I),I=1,3),(IW(I+1*I,I=1,IPV)
C650 FORMAT(48I ERROR RETURN FROM LA05AD BECAUSE THE FOLLOWING .3A1 DI/
C     1.15HS ARE DEPENDENT/(2015))
C650 FORMAT(48I ERROR RETURN FROM LA05A BECAUSE THE FOLLOWING .3A1
C     1.15HS ARE DEPENDENT/(2015))
   G=-5.
GO TO 720
700 IF(LP.GT.0)WRITE(LP,710)
C710 FORMAT(33H+ERROR RETURN FROM LA05AD BECAUSE )
C710 FORMAT(33H+ERROR RETURN FROM LA05A BECAUSE )
710 RETURN
END
SUBROUTINE LA05BD(A*IND,IA,N,IP,IW,W,G,B,TRANS)
SUBROUTINE LA05B (A*IND,IA,N,IP,IW,W,G,B,TRANS)
DOUBLE  PRECISION A(IA),B(N),AM,W(N),G,SMALL
         REAL  A(IA),B(N),AM,W(N),G,SMALL
LOGICAL TRANS
INTEGER*2 IND(IA*2),IW(N*4)
INTEGER   IND(IA*2),IW(N*4)
INTEGER IP(N*2)
COMMON/LA05DD/SMALL,LP,LENU,LENU,NCP,LRON,LCOL
C IP(I+1),IP(I+2) POINT TO START OF ROW/COLUMN I OF U.
C IW(I+1),IW(I+2) ARE LENGTHS OF ROW/COL I OF U.
C IW(3),IW(4) HOLD ROW/COL NUMBERS IN PIVOTAL ORDER.
COMMON/LA05D /SMALL,LP,LENU,LENU,NCP,LRON,LCOL
IF(G.LT.0.)GO TO 400
KLL=IA-LENU+1
IF(TRANS)GO TO 300
C
C MULTIPLY VECTOR BY INVERSE OF L
IF(LENU.LE.0.)GO TO 112
L1=IA+1
DO 110 KK=1,LENU
   K=L1-KK
   I=IND(K,1)
   IF(B(I).EQ.0.)GO TO 110
   J=IND(K,2)
   B(J)=B(J)+A(K)*B(I)
110 CONTINUE
112 DO 113 I=1,N
   W(I)=B(I)
113 B(I)=0.
C
C MULTIPLY VECTOR BY INVERSE OF U
N1=N+1
DO 140 II=1,N
   I=N1-II
   I=IW(I,3)
   AM=W(I)
   KP=IP(I,1)
   IF(KP.GT.0)GO TO 130
   KP=-KP
   IP(I+1)=KP
140 CONTINUE
NZ=IW(I+1)
KL=KP-1+NZ
K2=KP+1
DO 120 K=K2*KL
J=IND(K+2)
120 AM=AM-A(K)*B(J)
130 IF(AM.EQ.0.)GO TO 140
J=IND(KP+2)
B(J)=AM/A(KP)
KPC=IP(J+2)
KL=IW(J+2)+KPC-1
IF(KL.EQ.KPC)GO TO 140
K2=KPC+1
DO 135 K=K2*KL
I=IND(K+1)
135 IP(I+1)=-IABS(IP(I+1))
140 CONTINUE
GO TO 500

C C MULTIPLY VECTOR BY INVERSE OF TRANSPOSE OF U
300 DO 303 I=1,N
W(I)=B(I)
303 B(I)=0.
DO 315 II=1,N
I=IW(II+4)
AM=W(I)
IF(AM.EQ.0.)GO TO 315
J=IW(II+3)
KP=IP(J+1)
AM=AM/A(KP)
B(J)=AM
KL=IW(J+1)+KP-1
IF(KP.EQ.KL)GO TO 315
K2=KP+1
DO 310 K=K2*KL
I=IND(K+2)
310 W(I)=W(I)-AM*A(K)
315 CONTINUE
C C MULTIPLY VECTOR BY INVERSE OF TRANSPOSE OF L
IF(KLL.GT.IA)RETURN
DO 330 K=KLL+IA
  J=IND(K+2)
  IF(B(J).EQ.0.)GO TO 330
  I=IND(K+1)
  B(I)=B(I)+A(K)*B(J)
330  CONTINUE
  GO TO 500
C
400  IF(LP.GT.0)WRITE(LP,410)
C410  FORMAT(// 47H ERROR RETURN FROM LA05BD BECAUSE EARLIER ENTRY)
C      1,18H GAVE ERROR RETURN)
        FORMAT(// 47H ERROR RETURN FROM LA05B BECAUSE EARLIER ENTRY)
        1,18H GAVE ERROR RETURN)
500  RETURN
END
SUBROUTINE LA05CD(A,IND,IA,N,IP,IW,W,G,U,MM)
SUBROUTINE LA05C(A,IND,IA,N,IP,IW,W,G,U,MM)
C DOUBLE PRECISION A(IA),G,U,AM,W(N),SMALL,AU
   REAL A(IA),G,U,AM,W(N),SMALL,AU
C INTEGER*2 IND(IA),IW(N),IP(N)
C COMMON/LA05D0/SMALL,LP,LENL,LENU,NCP,LROW,LCOL
C COMMON/LA05D1/SMALL,LP,LENL,LENU,NCP,LROW,LCOL
IF(G.LT.0) GO TO 640
JM=MM
C MCP LIMITS THE VALUE OF NCP PERMITTED BEFORE AN ERROR RETURN RESULTS.
   MCP=NCP+20
C REMOVE OLD COLUMN
   LENU=LENU-IW(JM*2)
   KP=IP(JM*2)
   IM=IND(KP,1)
   KL=KP+IW(JM*2)-1
   IW(JM*2)=0
   DO 40 K=KP,KL
      I=IND(K,1)
      IND(K,1)=0
      KR=IP(I,1)
      NZ=IW(I,1)-1
      IW(I,1)=NZ
      KRL=KR+NZ
   DO 10 KM=KR,KRL
      IF(IND(KM,2).EQ.JM) GO TO 20
   10 CONTINUE
20 A(KM)=A(KRL)
   IND(KM,2)=IND(KRL,2)
   IND(KRL,2)=0
C INSERT NEW COLUMN
   DO 110 II=1,N
      I=IW(II,3)
      IF(I.EQ.IM)M=II
      IF(DABS(W(I))*LE.SMALL) GO TO 110
      IF(ABS(W(I))*LE.SMALL) GO TO 110
         LENU=LENU+1
         LAST=II
   110 CONTINUE
IF(LCOL+LENL.LT.IA)GO TO 50
C COMPRESS COLUMN FILE IF NECESSARY.
   IF(NCP.GE.MCP .OR. LENL+LENU.GE.IA)GO TO 600
C CALL LASEDA(IND,IP(1.2)*N+1W(1.2)+IA ++FALSE.)
   CALL LASEE (A+IND,IP(1.2)*N+1W(1.2)+IA ++FALSE.)
50  LCOL=LCOL+1
   NZ=IW(JM+2)
   IF(NZ.EQ.0)IP(JM+2)=LCOL
   IW(JM+2)=NZ+1
   IND(LCOL+1)=I
   NZ=IW(I+1)
   KPL=IP(I+1)+NZ
   IF(KPL.GT.LROW)GO TO 55
   IF(IND(KPL+2).EQ.0)GO TO 90
C NEW ENTRY HAS TO BE CREATED.
55  IF(LENL+LROW+NZ.LT.IA)GO TO 60
   IF(NCP.GE.MCP .OR. LENL+LENU+NZ.GE.IA)GO TO 600
C COMPRESS ROW FILE IF NECESSARY.
C CALL LAOSED(A+IND(1.2)+IP.,N+1W+IA ++TRUE.)
   CALL LAOSEE (A+IND(1.2)+IP.,N+1W+IA ++TRUE.)
60  KP=IP(I+1)
   IP(I+1)=LROW+1
   IF(NZ.EQ.0)GO TO 80
   KPL=KP+NZ-1
   DO 70 K=KP+KPL
    LROW=LROW+1
   A(LROW)=A(K)
   IND(LROW+2)=IND(K+2)
70   IND(K+2)=0
80   LROW=LROW+1
   KPL=LROW
C PLACE NEW ELEMENT AT END OF ROW.
90   IW(I+1)=NZ+1
   A(KPL )=W(I)
   IND(KPL+2)=JM
110  W(I)=0.
   IF(IW(IM+1).EQ.0 .OR. IW(JM+2).EQ.0 .OR. M.GT.LAST)GO TO 580
C C FIND COLUMN SINGULARS. OTHER THAN THE SPIKE. NON-SINGULARS ARE C MARKED WITH W(J)=1. ONLY IW(.3) IS REVISED AND IW(.4) IS USED C FOR WORKSPACE.
INS=M
M1=M
W(JM)=1.
DO 150 II=M,LAST
I=IWM(I,3)
J=IWM(I,4)
IF(W(J).EQ.0.)GO TO 140
KP=IP(I,1)
KL=KP+IWM(I,1)-1
DO 130 K=KP,KL
J=IND(K,2)
130  
W(J)=1.
IWM(INS,4)=I
INS=INS+1
GO TO 150
C PLACE SINGLETONS IN NEW POSITION.
140  
IWM(M1,3)=I
M1=M1+1
150 CONTINUE
C PLACE NON-SINGLETONS IN NEW POSITION.
IJ=M+1
DO 160 II=M1,LAST
IWM(I,3)=IWM(IJ,4)
160 IJ=IJ+1
C PLACE SPIKE AT END.
IWM(LAST,3)=I
C
C FIND ROW SINGLETONS, APART FROM SPIKE ROW. NON-SINGLETONS ARE MARKED
C WITH W(I)=2. AGAIN ONLY IWM(*,3) IS REVISED AND IWM(*,4) IS USED
C FOR WORKSPACE.
LAST1=LAST
JNS=LAST
W(IM)=2.
J=JM
DO 190 IJ=M1,LAST
II=LAST+M1-IJ
I=IWM(II,3)
IF(W(I).NE.2.)GO TO 180
K=IP(I,1)
IF(IJ.NE.LAST)J=IND(K,2)
KP=IP(J,2)
190 CONTINUE
KL=KP+IW(J,2)-1
IW(JNS,4)=I
JNS=JNS-1
DO 170 K=KP, KL
I=IND(K,1)
170 W(I)=2.
GO TO 190

180 IW(LAST1,3)=I
LAST1=LAST1-1
190 CONTINUE
DO 195 II=M1, LAST1
JNS=JNS+1
I=IW(JNS,4)
W(I)=3.
195 IW(II,3)=I

C
C DEAL WITH SINGLETON SPIKE COLUMN. NOTE THAT BUMP ROWS ARE MARKED BY
C
W(I)=3.
DO 220 II=M1, LAST1
KP=IP(JM,2)
KL=KP+IW(JM,2)-1
IS=0
DO 210 K=KP, KL
L=IND(K,1)
IF(W(L).NE.3.) GO TO 210
IF(IS.NE.0) GO TO 230
I=L
KNP=K
IS=1
210 CONTINUE
IF(IS.EQ.0) GO TO 580
C MAKE A(I,JM) A PIVOT.
IND(KNP,1)=IND(KP,1)
IND(KP,1)=I
KP=IP(I,1)
DO 215 K=KP, IA
IF(IND(K,2).EQ.JM) GO TO 216
215 CONTINUE
216 AM=A(KP)
A(KP)=A(K)
A(K)=AM
IND(K,2)=IND(KP,2)
IND(KP,2)=JM
JM=IND(K,2)
IW(I,4)=I
220 W(I)=2.
II=LAST1
GO TO 250
230 IN=M1
DO 240 IJ=II,LAST1
IW(IJ,4)=IW(IN,3)
240 IN=IN+1
LAST2=LAST1-1
IF(M1.EQ.LAST1)GO TO 485
DO 260 I=M1,LAST2
260 IW(I,3)=IW(I,4)
M1=II
IF(M1.EQ.LAST1)GO TO 485
C
C CLEAR W
DO 270 I=1,N
270 W(I)=0.
C
C PERFORM ELIMINATION
IR=IW(LAST1,3)
DO 480 II=M1,LAST1
IPP=IW(II,3)
KP=IP(IPP,1)
KR=IP(IR,1)
JP=IND(KP,2)
IF(KR.EQ.LAST1)JP=JM
C SEARCH NON-PIVOT ROW FOR ELEMENT TO BE ELIMINATED.
C AND BRING IT TO FRONT OF ITS ROW
KRL=KR+IW(IR,1)-1
DO 290 KNP=KR,KRL
IF(JP.EQ.IND(KNP,2))GO TO 300
290 CONTINUE
IF(II-LAST1)*480.*580.*480
C BRING ELEMENT TO BE ELIMINATED TO FRONT OF ITS ROW.
300 AM=A(KNP)
A(KNP)=A(KR)
A(KR)=AM
IND(KNP,2)=IND(KR,2)
IND(KR,2)=JP
IF(I1.EQ.LAST1)GO TO 310

C
IF(DABS(A(KP)) .LT. U*DABS(A(AM)))GO TO 310
IF(ABS(A(KP)) .LT. U*ABS(A(AM)))GO TO 310
C
IF(DABS(A(AM)) .LT. U*DABS(A(KP)))GO TO 330
IF(ABS(A(AM)) .LT. U*ABS(A(KP)))GO TO 330
IF(IW(IIPP,1).LE.IW(IR,1))GO TO 330

C PERFORM INTERCHANGE
310  IW(LAST1,3)=I1
      IW(I1,3)=IR
      IR=I1
      IPP=IW(I1,3)
      K=KR
      KR=KP
      KP=K
      KJ=JP(JP,2)
      DO 320 K=KJ,IA
      IF(IND(K,1).EQ.IPP)GO TO 325

320  CONTINUE
325  IND(K,1)=IND(KJ,1)
      IND(KJ,1)=IPP
      IF(A(KP).EQ.0)GO TO 580
      IF(I1.EQ.LAST1)GO TO 480
      AM=A(K)/A(KP)

C COMPRESS ROW FILE UNLESS IT IS CERTAIN THAT THERE IS ROOM FOR NEW ROW.
      IF(LROW+IW(IR,1)+IW(IIPP,1)+LENL.LE.IA)GO TO 340
      IF(NCP.GE.MCP.OR.LENU+IW(IR,1)+IW(IIPP,1)+LENL.GT.IA)GO TO 600
C
      CALL LA05E(A,IND(1,2),IP,N,IW,IA,.TRUE.)
      CALL LA05E(A,IND(1,2),IP,N,IW,IA,.TRUE.)
      KP=I1
      KR=IP(IR,1)

340  KRL=KR+IW(IR,1)-1
      KQ=KP+1
      KPL=KP+IW(IIPP,1)-1

C PLACE PIVOT ROW (EXCLUDING PIVOT ITSELF) IN W.
      IF(KQ.GT.IROW)GO TO 350
      DO 345 K=KQ,KPL
      J=IND(K,2)
      W(J)=A(K)
      IF(IR,1)=LROW+1
C TRANSFER MODIFIED ELEMENTS.
     IND(KR,2)=0
     KR=KR+1
     IF(KR.GT.KRL)GO TO 380
     DO 370 KS=KR,KRL
         J=IND(KS,2)
         AU=A(KS)+AM*W(J)
         IND(KS,2)=0
     C IF ELEMENT IS VERY SMALL REMOVE IT FROM U.
     C IF(DABS(AU).LE.SMALL)GO TO 365
     C IF(ABS(AU).LE.SMALL)GO TO 365
     G=DMAX1(G,DABS(AU))
     G=AMAX1(G,ABS(AU))
     LROW=LROW+1
     A(LROW)=AU
     IND(LROW,2)=J
     GO TO 370
 365 LENU=LENU-1
C REMOVE ELEMENT FROM COL FILE.
     K=IP(J,2)
     KL=K+I(W(J,2)-1
     I(W(J,2))=KL-K
     DO 366 KK=K,KL
         IF(IND(KK,1).EQ.IR)GO TO 367
     366 CONTINUE
 367 IND(KK,1)=IND(KL,1)
     IND(KL,1)=0
 370 W(J)=0.
C C SCAN PIVOT ROW FOR FILLS.
 380 IF(KQ.GT.KPL)GO TO 435
     DO 430 KS=KQ,KPL
         J=IND(KS,2)
         AU=AM*W(J)
     C IF(DABS(AU).LE.SMALL)GO TO 430
     C IF(ABS(AU).LE.SMALL)GO TO 430
         LROW=LROW+1
     A(LROW)=AU
     IND(LROW,2)=J
     LENU=LENU+1
C CREATE FILL IN COLUMN FILE.
   NZ=IW(J+2)
   K=IP(J+2)
   KL=K+NZ-1
C IF POSSIBLE PLACE NEW ELEMENT AT END OF PRESENT ENTRY.
   IF(KL.NE.LCOL)GO TO 390
   IF(LCOL+LENL.GE.IA)GO TO 400
   LCOL=LCOL+1
   GO TO 395
390   IF(IND(KL+1,1).NE.0)GO TO 400
395   IND(KL+1,1)=IR
   GO TO 425
C NEW ENTRY HAS TO BE CREATED.
400   IF(LCOL+LENL+NZ+1.LT.IA)GO TO 410
C COMPRESS COLUMN FILE IF THERE IS NOT ROOM FOR NEW ENTRY.
   IF(NCP.GE.MCP.OR. LENU+LENL+NZ+1.GE.IA)GO TO 600
C CALL LA05ED(A,IND,IP(1,2)*N,IW(1,2)*IA ..FALSE.)
   CALL LA05E (A,IND,IP(1,2)*N,IW(1,2)*IA ..FALSE.)
   K=IP(J+2)
   KL=K+NZ-1
C TRANSFER OLD ENTRY INTO NEW.
410   IP(J+2)=LCOL+1
   DO 420 KK=K,KL
   LCOL=LCOL+1
   IND(LCOL+1)=IND(KK,1)
420   IND(KK,1)=0
C ADD NEW ELEMENT.
   LCOL=LCOL+1
   IND(LCOL+1)=IR
C425  G=DMAX1(G,DABS(AU))
425  G=AMAX1(G, ABS(AU))
430   W(J)=0.
435   IW(IR+1)=LROW+1-IP(IR+1)
C C STORE MULTIPLIER
   IF(LENL+LCOL+1.LE.IA)GO TO 450
C COMPRESS COL FILE IF NECESSARY.
   IF(NCP.GE.MCP)GO TO 600
C CALL LA05ED(A,IND,IP(1,2)*N,IW(1,2)*IA ..FALSE.)
CALL LA05E (A, IND, IP(1:2), N, IW(1:2), IA ..FALSE.)
K=IA-LENL
LENL=LENL+1
A(K)=AM
IND(K,1)=IPP
IND(K,2)=IR
C CREATE BLANK IN PIVOTAL COLUMN.
KP=IP(JP,2)
NZ=IW(JP,2)-1
KL=KP+NZ
DO 460 K=KP, KL
   IF(IND(K,1).EQ.IR)GO TO 465
460 CONTINUE
465 IND(K,1)=IND(KL,1)
   IW(JP,2)=NZ
   IND(KL,1)=0
   LENU=LENU-1
CONTINUE
C
C CONSTRUCT COLUMN PERMUTATION AND STORE IT IN IW(1:4)
DO 490 II=M, LAST
   I=IW(II,3)
   K=IP(I,1)
   J=IND(K,2)
490 IW(II,4)=J
GO TO 720
C
C THE FOLLOWING INSTRUCTIONS IMPLEMENT THE FAILURE EXITS.
IF(LP.NE.0)WRITE(LP,590)
590 FORMAT(//34X,45HSINGULAR MATRIX CREATED BY REPLACEMENT OF COL.15)
   G=-6.
   GO TO 700
IF(LP.NE.0)WRITE(LP,610)
610 FORMAT(//34X,15HIA IS TOO SMALL)
   G=-7.
   GO TO 700
IF(LP.NE.0)WRITE(LP,650)
650 FORMAT(//34X,31HEARLIER ENTRY GAVE ERROR RETURN)
IF(LP.NE.0)WRITE(LP,710)
710 FORMAT(33H+ERROR RETURN FROM LA05CD BECAUSE )
DI/
C710 FORMAT(33H+ERROR RETURN FROM LA05C BECAUSE )
RETURN
END

BLOCK DATA
COMMON/LA05DD/SALL,LP,LENL,LENUL,NCPL,LRW0,LCOL
COMMON/LA05U/SALL,LP,LENL,LENUL,NCPL,LRW0,LCOL
C  DOUBLE PRECISION SMALL
REAL SMALL
C  DATA SMALL,LP/0.0,0.6/
DATA SMALL,LP/0.0,6/
END
SUBROUTINE LA05ED(A,IRN,IP,N,IW,IA,REALS)
SUBROUTINE LA05E(A,IRN,IP,N,IW,IA,REALS)
LOGICAL REALS
DOUBLE PRECISION A(IA),SMALL
REAL A(IA)
INTEGER*2 IRN(IA),IW(N)
INTEGER IRN(IA),IW(N)
INTEGER IP(N)
COMMON/LA05DD/SMALL,LP,LENL,LENU,NCP,LRW,LCOL
COMMON/LA05D/SMALL,LP,LENL,LENU,NCP,LRW,LCOL
NCP=NCP+1

COMPRESSION FILE OF POSITIVE INTEGER. ENTRY J STARTS AT IRN(IP(J))
AND CONTAINS IW(J) INTEGERS, J=1..N. OTHER COMPONENTS OF IRN ARE ZERO.
LENGTH OF COMPRESSED FILE PLACED IN LRW IF REALS IS .TRUE. OR LCOL
OTHERWISE.
IF REALS IS .TRUE. ARRAY A CONTAINS A REAL FILE ASSOCIATED WITH IRN
AND THIS IS COMPRESSED TOO.
A,IRN,IP,IW,IA ARE INPUT/OUTPUT VARIABLES.
N,REALS ARE INPUT/UNCHANGED VARIABLES.

DO 5 J=1,N
C STORE THE LAST ELEMENT OF ENTRY J IN IW(J) THEN OVERWRITE IT BY -J.
  NZ=IW(J)
  IF(NZ.LE.0)GO TO 5
  K=IP(J)+NZ-1
  IW(J)=IRN(K)
  IRN(K)=-J
5 CONTINUE
C KN IS THE POSITION OF NEXT ENTRY IN COMPRESSED FILE.
  KN=0
  IP=0
  KL=LCOL
  IF(REALS)KL=LRW
C LOOP THROUGH THE OLD FILE SKIPPING ZERO (DUMMY) ELEMENTS AND
C MOVING GENUINE ELEMENTS FORWARD, THE ENTRY NUMBER BECOMES
C KNOWN ONLY WHEN ITS END IS DETECTED BY THE PRESENCE OF A NEGATIVE
C INTEGER.
DO 25 K=1,KL
  IF(IRN(K).EQ.0)GO TO 25
  KN=KN+1
  IF(REALS)A(KN)=A(K)
IF(IRN(K).GE.0)GO TO 20
C END OF ENTRY. RESTORE IRN(K), SET POINTER TO START OF ENTRY AND
C STORE CURRENT KN IN IPI READY FOR USE WHEN NEXT LAST ENTRY
C IS DETECTED.
J=-IRN(K)
IRN(K)=IW(J)
IP(J)=IPI+1
IW(J)=KN-IPI
IPI=KN
20 IRN(KN)=IRN(K)
25 CONTINUE
IF(REALS)LROW=KN
IF(.NOT.REALS)LCOL=KN
RETURN
END
References


