1. (a). Let $\mathbf{x} \in \mathbb{R}^n$. What is a linesearch method for the unconstrained minimization of the objective function $f(\mathbf{x})$? What is the Armijo condition and why is it important? Briefly describe the backtracking-Armijo linesearch method.

[8 marks]

(b). What are the steepest-descent and Newton directions? Give one advantage and one disadvantage of each direction when used for unconstrained minimization.

[6 marks]

(c). Suppose $f(\mathbf{x}) = x_1^4 - 2x_1^2 + x_2^2$. Why is the Newton direction at $\mathbf{x} = (\frac{1}{2}, 1)$ unsuitable for unconstrained minimization. How might it be modified to give a suitable direction?

[6 marks]

2. Consider the problem

$$\underset{x \in \mathbb{R}^n}{\text{minimize}} \;\; f(\mathbf{x}) \;\; \text{subject to} \;\; \mathbf{c}(\mathbf{x}) \geq 0 \tag{2}$$

where $f : \mathbb{R}^n \to \mathbb{R}$ and $\mathbf{c} : \mathbb{R}^n \to \mathbb{R}^m$.

(a) Write down first-order necessary conditions for $\mathbf{x}_*$ to be a solution to the problem (2). [3 marks]

(b) What is the logarithmic barrier function for (2). Write down the gradient of this barrier function. [2 marks]

(c) Using your solutions to (a) and (b), briefly explain why you would expect a minimizer of the logarithmic barrier function to converge to a first-order critical point of the problem (2) as the barrier parameter is reduced towards zero. State any assumptions that you need to make. Explain how the barrier problem leads to Lagrange multiplier estimates for (2). [7 marks]

Now suppose that $n = 2$, $m = 1$, $f(\mathbf{x}) = \frac{1}{2}x_1^2 + x_2$ and $\mathbf{c}(\mathbf{x}) = x_2$ in (2)..

(d) What is the solution of the problem (2) in this case, and what is its Lagrange multiplier? [2 marks]

(e) By considering the Hessian matrix of the barrier function in this case, give one reason why might you expect that the barrier subproblems might become successively harder to solve as the barrier parameter is reduced to zero? [3 marks]

(f) Indicate why the difficulty you described in (e) does not actually happen in practice. [3 marks]